Fuzzy Entropy Image Segmentation Based on a Modified Backtracking Search Optimization Algorithm

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***Abstract*— In this paper, a multilevel thresholding algorithm based on backtracking search optimization algorithm (BSA) is introduced. The simple structure of BSA which is effective and fast enables it to easily adapt to threshold selection problem. To enhance the local search ability of BSA, a classical differential evolution (DE) algorithm and an improved BGA (IBGA) mutation are used to work with BSA. The modified algorithm evaluates the quality of the thresholds under the fitness function which is generated using fuzzy entropy method. The threshold combination with the best fitness value is the optimal solution to the multilevel segmentation task. Experimental results demonstrate the effectiveness of the proposed method. Besides, the proposed method is employed to segment magnetic resonance images of brain and the segmentation results also indicates the capability of the new approach.**

Keywords—multilevel thresholding; backtracking search optimization algorithm; differential evolution algorithm; fuzzy entropy; magnetic resonance image

I. INTRODUCTION

Segmentation is one of the most important steps in image processing. Its aim is to group pixels into separate regions which correspond to different objects [1]. Many different segmentation techniques have been created and developed and these techniques can be classified into 6 categories: threshold based method, region based method, edge based method, fuzzy theory based method, graph cuts based method and machine learning based method [2].

Thresholding is one of the easiest methods when segmenting an image. A threshold is given and pixels whose intensity value is higher than this threshold are labeled as first class while the rest correspond to a second class [3]. When pixels are expected to be separated into more than two classes, multilevel thresholding will be applied and more than one threshold values are needed [4].

There are two widely used thresholding methods. Over years of research, these two methods have been proved to be effective and efficient [5]. The first method is proposed by Otsu in 1979 [6]. Otsu's method is a nonparametric technique and its idea is to maximize the variance between classes. The second nonparametric thresholding method is Kapur's method [7]. This method aims to find the optimal threshold values that maximize the overall entropy which measures the compactness and separability among classes. In this sense, when the overall entropy is optimized, the thresholds which can separate the image appropriately are obtained.

Apart from the methods mentioned above, methods based on the concept of fuzzy entropy [8] have been popular in recent years. Fuzzy entropy is an extension of Shannon's definition and its meaning is quite different from the classical Shannon entropy due to the fact that fuzzy entropy contains vagueness and ambiguity uncertainties, while Shannon entropy contains randomness uncertainty [9]. Fuzzy entropy image segmentation, which takes the fuzziness and uncertainty of the image into account, has had great success in image thresholding segmentation [10] [11] [12].

As an alternative to classic optimization methods, evolutionary algorithms (EA) have been used to handle multilevel thresholding problems [13]. Genetic algorithms (GA) [14], inspired on the biological evolution, have been used to solve segmentation tasks. Reference [15] proposed an algorithm based on GA for multilevel thresholding and the results are quite promising. Particle swarm optimization (PSO) [16], artificial bee colony (ABC) [17] is a kind of EA inspired on swarm behavior. In [18], both algorithms are combined with Kapur's method to find the optimal multilevel threshold values. In [19], bacterial foraging algorithm (BFA) is employed to maximize the Kapur's and Otsu's objective functions and show competitive results. Harmony search algorithm (HSA) [20] is a new kind of EA which is based on the process when musicians search for a better state of harmony. Its simple structure and fast convergence rate draw great attention and many work has been done to handle segmentation tasks [21] [22].

In this paper, a new EA, backtracking search optimization algorithm (BSA) [23], is combined with fuzzy entropy theory to solve multilevel thresholding problems. BSA is of a simple structure and shows great advantages when solving complex optimization problems. It has great global exploration ability as well as good local exploitation ability. To further enhance its local exploitation ability, a classical differential evolution (DE) algorithm [25] and an improved BGA (IBGA) mutation are applied to work with BSA together. Experimental results on benchmark images indicate the effectiveness of the proposed method in terms of accuracy and robustness.

The proposed multithresholding segmentation algorithm is employed to solve the segmentation task of magnetic resonance (MR) images of brain. Original MR images and the segmentation results of these images are presented and it is obvious that the proposed algorithm can handle MRI segmentation tasks.

The rest of the paper is organized as follows. Section 2 provides a brief introduction of BSA and fuzzy entropy theory. Section 3 presents a detailed description of the proposed method. Experimental and comparison results as well as analysis are presented in section 4. Section 5 includes an application in magnetic resonance imaging segmentation. At the end, section 6 concludes this paper.

II. PRELIMINARIES

A. Backtracking Search Optimization Algorithm

BSA is a new evolution algorithm (EA) proposed by Pinar Civicioglu in 2013. Unlike many other intelligent algorithms, BSA has only one control parameter and the quality of solutions is not sensitive to the initial value of this parameter. Because of its simple and effective structure, BSA can be used to solve many complex optimization problems. BSA has five processes just like other EAs.

Initialization. BSA initializes the population P with (1) where U is the uniform distribution and *low*, *up* are the lower and upper bounds respectively.

$$P_{i,j} \sim U(low_j, up_j) \quad (1)$$

for $i=1,2,\ldots,N$ and $j=1,2,\ldots,D$, where N and D are the population size and the problem dimension.

Selection-I. BSA's Selection-I stage determines the historical population *oldP* according to (2) and redefines it at the beginning of each generation using (3) where $a, b \sim U(0, 1)$ is satisfied.

$$oldP_{i,j} \sim U(low_j, up_j) \quad (2)$$

$$\text{oldP}=\begin{cases} P & ,a<b \\ oldP & ,otherwise \end{cases} \quad (3)$$

The permuting function in (4) is a random shuffling function and it is used to randomly change the order of the individuals $oldP_i$ in oldP.

$$oldP := permuting(oldP) \quad (4)$$

Mutation. BSA employs a random mutation strategy to generate the initial form of the trial population *Mutant* using (5):

$$Mutant=P+scale\_factor\cdot(oldP-P) \quad (5)$$

BSA uses the historical population in the calculation of the search-direction matrix where *scale_factor* controls the amplitude of the search-direction matrix. In this paper, $scale\_factor=3\cdot randn$ where $randn \sim N(0,1)$.

Crossover. Trial individuals with better fitness values are used to evolve the target population individuals. BSA calculates a binary integer-valued matrix (*map*) to indicate the individuals of *Mutant* to be manipulated using the relevant individuals of P. Equation (6) shows BSA's crossover strategy.

$$Mutant_{i,j}=\begin{cases} P_{i,j} & , \quad map_{i,j}=1 \\ Mutant_{i,j} & , otherwise \end{cases} \quad (6)$$

Selection-II. At this stage, a greedy selection mechanism is employed to select and update the population to be used in the next generation. This process is shown as (7).

$$P_i^{next}=\begin{cases} Mutant_i & , f(Mutant_i) \le f(P_i) \\ P_i & , \quad otherwise \end{cases} \quad (7)$$

B. Fuzzy Entropy

The measure of uncertainty is adopted as a measure of information and the measure of quality of fuzzy information is known as fuzzy entropy [26]. The meaning of fuzzy entropy is quite different from that of the classical Shannon entropy as it takes fuzziness and uncertainty into account.

Fuzzy entropy is defined under the concept of membership function. In this paper, to handle n-level thresholding problems, n membership functions, $\mu_1, \mu_2, \ldots, \mu_n$ are used. These n membership functions are in the following form [33]:

$$\mu_1(k)=\begin{cases} 1 & , \quad k \le a_1 \\ \frac{k-c_1}{a_1-c_1} & , \ a_1 \le k \le c_1 \\ 0 & , \quad k > c_1 \end{cases} \quad (8)$$

$$\vdots$$

$$\mu_{n-1}(k)=\begin{cases} 0 & , \ k \le a_{n-2} \\ \frac{k-a_{n-2}}{c_{n-2}-a_{n-2}} & , a_{n-2} \le k \le c_{n-2} \\ 1 & , \ c_{n-2} \le k \le a_{n-1} \\ \frac{k-c_{n-1}}{a_{n-1}-c_{n-1}} & , a_{n-1} \le k \le c_{n-1} \\ 0 & , \ k > c_{n-1} \end{cases} \quad (9)$$

$$\mu_n(k) = \begin{cases} 0\ , & k \le a_{n-1} \\ \frac{k-a_{n-1}}{c_{n-1}-a_{n-1}}\ , & a_{n-1} \le k \le c_{n-1} \\ 1\ , & k > c_{n-1} \end{cases} \quad (10)$$

where these 2*(n-1) fuzzy parameters, $a_{1,}c_1\ ...\ a_{n-1}, c_{n-1}$, are unknown and $0 \le a_1 \le c_1 \le \cdots \le a_{n-1} \le c_{n-1} \le L-1$.

The fuzzy entropy of each segmentation of the n-level thresholding problem can be defied as

$$FE_1 = -\sum_{k=0}^{L-1} \frac{p_k * \mu_1(k)}{P_1} * \ln\left(\frac{p_k * \mu_1(k)}{P_1}\right) \quad (11)$$

$$FE_2 = -\sum_{k=0}^{L-1} \frac{p_k * \mu_2(k)}{P_2} * \ln\left(\frac{p_k * \mu_2(k)}{P_2}\right) \quad (12)$$

$$\vdots$$

$$FE_n = -\sum_{k=0}^{L-1} \frac{p_k * \mu_n(k)}{P_n} * \ln\left(\frac{p_k * \mu_n(k)}{P_n}\right) \quad (13)$$

where $P_1 = \sum_{k=0}^{L-1} p_k * \mu_1(k), P_2 = \sum_{k=0}^{L-1} p_k * \mu_2(k) \ldots P_n = \sum_{k=0}^{L-1} p_k * \mu_n(k)$. The total fuzzy entropy is calculated using

$$\mathrm{FE} = FE_1 + FE_2 + \cdots + FE_n \quad (14)$$

and the optimal combination of parameters can be obtained by maximizing FE

$$\varphi(a_{1,}c_1\ ...\ a_{n-1}, c_{n-1}) = argmax(\ FE\ ) \quad (15)$$

III. Modified BSA for Fuzzy Entropy Image Segmentation

A. Modified BSA

Comparison results have shown BSA's success in solving numerical optimization problems [23]. Its simple structure enables it to benefit from previous generation population and the global exploration ability as well as local exploitation ability make the convergence fast and accurate.

However, in some practical research [28], BSA can't provide enough local search ability and therefore, some tricks have to be done to make this up. In this paper, inspired by the idea of Zhao [28], a classical DE algorithm is adopted to enhance the local exploitation ability of BSA. The DE mutation strategy we use is called a "current-to-best/1" mutation and is introduced by (11):

$$Mutant_i = P_i + F \times (\ P_{best} - P_i\ ) + F \times (\ P_{r1} - P_{r2}\ ) \quad (16)$$

In (16), r_1, r_2 are integers randomly selected from {1, 2. . . *N*} and $r_1 \ne r_2 \ne r_3$. The factor *F* is a number between 0 and 1. P_i is the current individual in the population called target vector and P_{best} is the best individual in the current population. $Mutant_i$ is called a mutant vector. After mutation, both P_i and $Mutant_i$ are used in the crossover operation to generate a trial vector P_i using (17):

$$P_{i,j} = \begin{cases} Mutant_{i,j}\ , & if\ rand\ \le Cr\ or\ j = j_{rand} \\ P_{i,j}\ , & otherwise \end{cases} \quad (17)$$

where $i \in \{1,2,\ldots,N\}$, $j \in \{1,2,\ldots,D\}$. C_r is the crossover control parameter and j_{rand} is a randomly selected integer from 1 to *D*. $P_{i,j}$ is the j_{th} component of the trial vector $\overrightarrow{P_i}$.

To further promote the local search ability of BSA, an improved BGA (IBGA) mutation [24] is also applied. The IBGA is defined as follows:

$$p_i' = \begin{cases} p_i \pm range_i \cdot \alpha\ , & i = rand \\ p_i\ , & otherwise \end{cases} \quad (18)$$

$$\alpha = \sum_{m=0}^{15} \alpha_m 2^{-m} \quad (19)$$

$$range_i = (up_i - low_i) \cdot rand(0, (1 - \frac{gen}{MaxGen})^7) \quad (20)$$

where *gen* is the current generation number and *MaxGen* is the total number of generations. Each α_m is set to be 0 initially and then mutated to 1 with probability $\mathrm{p}(\alpha_m = 1) = 1/16$. It is obvious that IBGA can easily get a neighborhood P_i' near an individual P_i and that will promote the local search ability of BSA.

B. Framework of Modified BSA for Fuzzy Entropy Image Segmentation

The framework of modified BSA for fuzzy entropy image segmentation is shown as follows:

Step 0. Set *gen* = 0.

Step 1. Initialize parameters (population size *N*, generation number *MaxGen*, threshold parameter *Thresh*, *scale_factor* and *DIM_RATE* in BSA, *F* and crossover rate C_r in DE)

Step 2. Generate population *P* and *oldP* using (1) and (2).

Step 3. Evaluate population *P* using (14).

Step 4. if *gen* < *Thresh*MaxGen (Here, Thresh is a flag to decide whether BSA or DE is executed)*, redefine *oldP* using (3) and (4). Perform mutation and crossover of BSA using (5) and (6). Execute IBGA according to (18), (19) and (20).

else perform mutation and crossover of DE using (16) and (17).

Step 5. Evaluate the fuzzy entropy value of the population *P* according to (14). Select the best individual and update population *P* for the next generation using (7).

Step 6. *gen* = *gen* +1. If *gen* < *MaxGen*, goto Step 4.

Step 7. Output

In Step 4, the whole evolutionary process is divided into two phases. In the first phase, BSA's mutation strategy is applied. This phase takes advantage of the experiences and information gained in the previous generation and the good global search ability of BSA is fully used to generate new population. In addition, IBGA is employed to improve the local exploit ability of BSA. In the second phase, DE's "current-to-best/1" mutation strategy is used. Since this mutation strategy exploits the information of the best individual in the current generation, it can guide the new population towards the global optimum. Therefore, the second phase can make up for the local exploit ability of BSA and the convergence velocity is improved.

IV. Experiments and Comparison

A. *Experimental Results*

In order to validate the effectiveness of the proposed algorithm, 6 benchmark images [29] [30] are selected as experimental data. All the images have the same size (512×512 pixels), and they are in JPEG format. The number of thresholds used in this test is set to be k=2, 3, 4, 5. For each test image, 30 independent runs are performed. To do quantitative analysis on the experimental results, three measure criteria, standard deviation (STD), feature similarity index (FSIM) [32] and peak-to-signal ratio (PSNR), are computed. STD represents the stability of the proposed algorithm. FSIM is computed using phase congruency (PC) and image gradient magnitude (GM) while PSNR evaluates the similarity of the segmented image and the original image based on mean square error (MSE). The parameter setting of the proposed method is shown in Table Ⅰ.

The segmentation results on 6 test images are presented in Table Ⅱ. It is obvious that the proposed segmentation method can get the proper thresholds of all 7 images effectively and the stability is quite good.

TABLE I. Parameter Value

	Parameters						
	N	***MaxGen***	***F***	C_r	***scale_factor***	***DIM_RATE***	***Thresh***
value	100	200	0.5	0.7	3*randn	1	2/3

B. *Comparisons With Other Segmentation Techniques*

Two sets of comparison experiments are executed to analyze the performance of the proposed approach. The first set is between four methods: the proposed method, fuzzy entropy with DE algorithm, fuzzy entropy with harmony search (HS) algorithm and fuzzy entropy with unmodified BSA. This set of comparison is to show the advantage of the modified BSA when fuzzy entropy is used in multilevel thresholding. The second set is executed between the proposed method and two Kapur-based segmentation techniques with PSO [31] and BFA [19] respectively. The comparison results are presented in Table Ⅳ and Table Ⅴ.

As shown in Table Ⅳ and Table Ⅴ, the performance of the proposed method is better than the comparison algorithms. Table Ⅳ shows that the modified BSA can get better thresholds than DE or HS according to PSNR and FSIM when fuzzy entropy theory is used in multilevel thresholding. Table Ⅴ provides evidence that the proposed approach is competitive when compared with two state-of-the-art thresholding methods. Both PSNR and FSIM are better in comparison with BFA and PSO.

TABLE II. Segmentation results of the proposed method

Image	***k***	***Thresholds***	***PSNR***	***STD***
Baboon	2	52,179	10.386	3.6E-15
	3	54,107,181	16.365	8.6E-02
	4	35,85,117,190	17.400	2.3E-01
	5	34,75,111,142,194	20.699	2.9E-01
Bridge	2	64,192	12.969	1.9E-15
	3	69,138,197	16.598	1.4E-02
	4	40,106,153,214	17.378	3.7E-01
	5	39,88,132,173,220	19.863	3.9E-01
Living room	2	67,194	11.799	1.8E-15
	3	45,118,201	15.882	8.3E-02
	4	44,98,139,211	19.142	9.5E-02
	5	41,87,125,167,217	20.502	1.8E-01
Lena	2	74,202	11.113	1.8E-15
	3	68,135,195	16.962	1.5E-15
	4	60,117,146,210	18.733	1.2E-01
	5	108,139,170,213	21.295	1.5E-01
Peppers	2	58,185	11.810	1.8E-15
	3	60,120,188	17.047	2.8E-03
	4	36,97,136,200	17.365	1.0E-01
	5	36,72,108,143,199	20.016	1.6E-01
Camera man	2	93,220	12.897	1.8E-15
	3	43,135,220	13.7527	5.6E-01
	4	48,95,143,220	19.717	1.1E-01
	5	48,96,146,194,227	20.043	3.3E-01

TABLE III. Segmentation results for MR images of brain

Original image	***Segmented image***	***White matter***	***Gray matter***	***Cerebrospinal fluid***

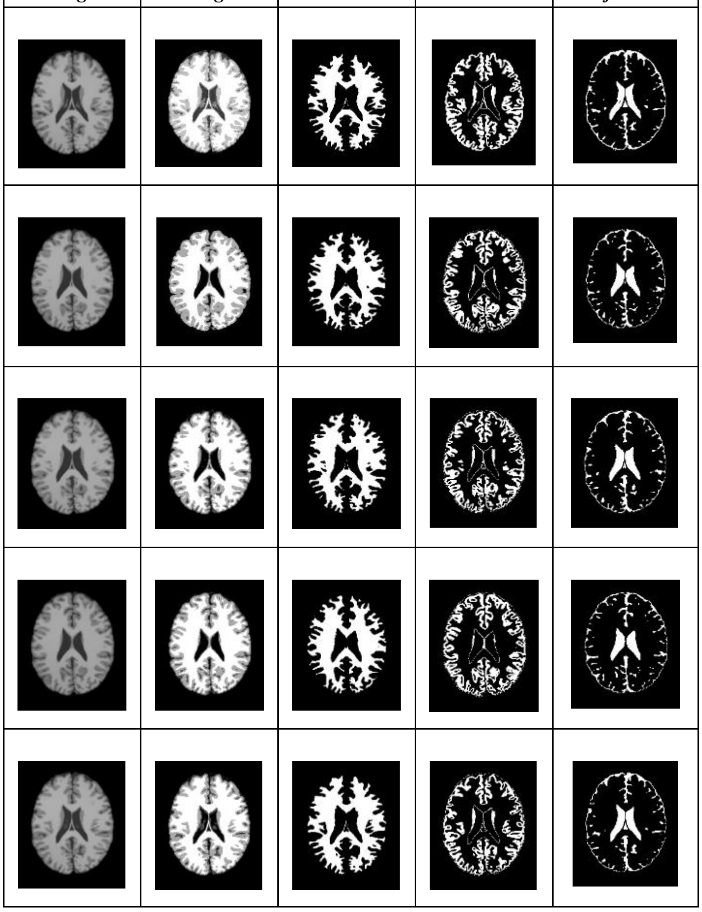

TABLE IV. COMPARISON RESULTS WITH OTHER METHODS USING FUZZY ENTROPY

Image	***k***	***PSNR***				***FSIM***			
		Proposed	***DE***	***HS***	***BSA***	***Proposed***	***DE***	***HS***	***BSA***
Baboon	2	**10.386**	10.386	10.386	10.386	**0.553**	0.553	0.553	0.553
	3	**16.365**	16.362	15.533	16.334	**0.803**	0.803	0.781	0.799
	4	17.400	17.315	**17.797**	17.335	0.821	0.818	**0.844**	0.814
	5	**20.699**	20.664	20.617	20.617	**0.916**	0.902	0.911	0.911
Bridge	2	**12.969**	12.969	12.958	12.969	**0.687**	0.683	0.684	0.687
	3	**16.598**	16.568	16.191	16.417	**0.840**	0.838	0.827	0.839
	4	17.378	17.407	**17.749**	17.650	0.868	0.869	**0.871**	0.864
	5	**19.863**	19.739	19.592	19.812	**0.915**	0.913	0.902	0.914
Living room	2	**11.799**	11.799	11.625	11.799	**0.625**	0.625	0.620	0.625
	3	**15.882**	15.826	15.876	15.827	**0.754**	0.751	0.750	0.752
	4	**19.142**	19.139	18.735	19.091	**0.840**	0.838	0.834	0.839
	5	20.502	**20.518**	20.094	20.447	**0.889**	0.886	0.879	0.887
Lena	2	**11.113**	11.113	11.105	11.112	**0.629**	0.629	0.626	0.623
	3	**16.962**	16.962	16.931	16.913	**0.759**	0.759	0.750	0.759
	4	18.733	18.721	**19.201**	18.769	0.793	0.786	**0.796**	0.786
	5	**21.295**	21.231	20.975	21.222	**0.849**	0.843	0.849	0.834
Peppers	2	**11.810**	11.810	11.807	11.810	**0.647**	0.647	0.647	0.647
	3	**17.047**	17.043	16.445	17.044	**0.781**	0.781	0.775	0.781
	4	**17.365**	17.312	17.306	17.353	**0.811**	0.802	0.805	0.811
	5	**20.016**	20.010	19.911	20.014	**0.868**	0.865	0.849	0.862
Camera man	2	**12.897**	12.897	12.897	12.897	**0.739**	0.732	0.739	0.729
	3	13.753	13.646	**15.791**	13.737	0.758	0.727	**0.759**	0.744
	4	**19.717**	19.517	19.374	19.635	**0.847**	0.813	0.831	0.844
	5	20.043	19.996	**21.223**	20.041	0.841	0.839	**0.849**	0.845

TABLE V. COMPARISON RESULTS WITH TWO ALGORITHMS USING KAPUR'S METHOD

Image	***k***	***PSNR***			***FSIM***		
		Proposed	***BFA***	***PSO***	***Proposed***	***BFA***	***PSO***
Baboon	2	10.386	10.386	**13.479**	0.553	0.553	**0.564**
	3	**16.365**	16.015	15.875	**0.803**	0.737	0.701
	4	17.400	17.441	**18.566**	0.821	0.793	**0.829**
	5	**20.699**	18.924	19.954	**0.916**	0.893	0.907
Bridge	2	**12.969**	10.915	10.671	**0.687**	0.665	0.648
	3	**16.598**	14.296	14.265	**0.840**	0.801	0.799
	4	**17.378**	16.112	15.057	**0.868**	0.796	0.785
	5	**19.863**	17.678	15.883	**0.915**	0.896	0.887
Living room	2	**11.799**	11.062	11.682	**0.625**	0.601	0.619
	3	**15.882**	15.829	13.701	**0.754**	0.731	0.698
	4	**19.142**	16.699	17.752	**0.840**	0.761	0.772
	5	**20.502**	17.993	19.257	**0.889**	0.803	0.825
Lena	2	11.113	**12.632**	10.633	0.629	**0.632**	0603
	3	**16.962**	15.439	13.246	**0.759**	0.729	0.698
	4	**18.733**	16.578	16.949	**0.793**	0.727	0.752
	5	**21.295**	17.199	17.326	**0.849**	0.684	0.690
Peppers	2	11.810	**14.593**	10.507	0.647	**0.751**	0.629
	3	**17.047**	15.969	15.672	**0.781**	0.746	0.750
	4	**17.365**	17.104	15.900	**0.811**	0.803	0.755
	5	**20.016**	18.745	18.828	**0.868**	0.820	0.834
Camera man	2	**12.897**	9.1954	8.675	**0.739**	0.651	0.618
	3	**13.753**	12.010	10.853	**0.758**	0.737	0.694
	4	**19.717**	15.034	17.846	**0.847**	0.590	0.819
	5	**20.043**	17.012	17.929	**0.841**	0.798	0.821

V. APPLICATION IN MAGNETIC RESONANCE IMAGING SEGMENTATION

Magnetic resonance imaging (MRI) is a medical imaging technique used in radiology to investigate the anatomy and physiology of the body in both health and disease. MRI scanners use magnetic fields and radio waves to form images of the body. The technique is widely used in hospitals for medical diagnosis, staging of disease and for follow-up without exposure to ionizing radiation. Accurate segmentation of MR images of brain is of great importance for the study and the treatment of various pathologies such as Alzheimer disease, Parkinson or Parkinson related syndrome.

In this paper, the proposed method is used to segment MR images of brain. These MR images come from the Brain Web database of Mcgill University [27] and the parameter setting is: T1 modality, 1mm slice thickness, 0% noise and 20% intensity non-uniformity. The task is to segment each image into 3 parts: white matter, gray matter and cerebrospinal fluid. Table Ⅲ shows the results of the segmentation.

VI. CONCLUSIONS

In this paper, a multilevel thresholding segmentation method based on the concept of fuzzy entropy is presented. The proposed method employs a modified BSA to serve as the optimization algorithm and the outstanding global and local search ability enable the method to get the appropriate thresholds of the segmentation task. To demonstrate the effectiveness of the proposed method, two sets of experiments are carried out. Both sets of experiment show evidence that the proposed approach is competitive when compared with other segmentation techniques which use PSO and BFA as optimization algorithms. The results of MRI segmentation application also indicate the capacity of the proposed approach. However, computation time of the proposed method still has room to improve and that will be investigated in future work.

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