

# Fingerprint Image Segmentation Method Based on MCMC&GA\*

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**Abstract.** Fingerprint image segmentation is one key step in Automatic Fingerprint Identification System (AFIS), and how to do it faster, more accurately and more effectively is important for AFIS. This paper introduces the Markov Chain Monte Carlo (MCMC) method and the Genetic Algorithm (GA) into fingerprint image segmentation and brings forward a fingerprint image segmentation method based on Markov Chain Monte Carlo and Genetic Algorithm (MCMC&GA). Firstly, it generates a random sequence of closed curves, which is regarded as the boundary between the fingerprint image region and the background image region, as Markov Chain, which uses boundary curve probability density function (BCPDF) as the index of convergence. Then, it is simulated by Monte Carlo method with BCPDF as a parameter, which is converged at the maximum. Lastly, Genetic Algorithm is introduced to accelerate the convergent speed. In conclusion, the closed curve with the maximum value of the BCPDF is the ideal boundary curve. The experimental results indicate that the method is robust to the low-quality finger images.

## 1 Introduction

In recent years, the technology of Automatic Fingerprint Identification has caused people's extensive concerns [1~5]. Fingerprint image segmentation is a key problem in fingerprint image processing and it is also one of the most intensively studied areas in fingerprint identification system. It is important for AFIS that the fingerprint image is segmented faster, more accurately and effectively.

The present fingerprint image segmentation methods can be summed up two specials: one is based on block-level [2,3], the other is based on pixel-level [4,5]. Both designed the algorithms according to the statistical character (e.g. Variance, Mean) of the gray fingerprint image. Yin Y.L. et al used the model of quadratic curve surface to carry out the fingerprint image segmentation [6], which regarded the gray variance,

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the gray mean and the orientation coherence as the parameters of the model. Generally, it satisfies the demand of the fingerprint image segmentation processing in common cases, but the result is unsatisfied when the fingerprint images are stronger peeled or have stronger interference by the prior remainder image.

Over the past 40 years, Markov Chain Monte Carlo (MCMC) method has penetrated many subjects, such as statistical physics, seismology, chemistry, biometrics and protein folding, as a general engine for inference and optimization [7]. In computer vision, Zhu S C et al. has done many works but not fingerprint image [8,9,10]. He Y L et al regarded the fingerprint image as Markov Random Field and carried out the fingerprint image segmentation successfully [11]. But, it can only generate the boundary curve only where the edge contrast between fingerprint and background is stronger, and it is unsatisfied when the interference of background is stronger.

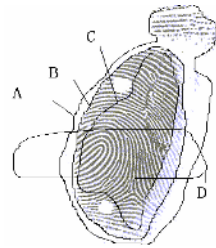
The paper takes the closed curve as research object and proposes the fingerprint image segmentation method based on MCMC&GA. Firstly, it randomly generates Markov chain of closed curves. Then, it is simulated by Monte Carlo method to converge at the boundary curve that has the biggest value of the boundary curve probability density function. Lastly, Genetic Algorithm is introduced to accelerate the convergent speed.

## 2 Fingerprint Boundary Curve Markov Chain

### 2.1 The Model of Boundary Curve Probability

To a fingerprint image, if the closed curve, which can separate the fingerprint image region from background image region, can be found out, we think that the fingerprint image will be segmented well. We call this closed curve as boundary curve. As Fig.1 showed, the curve B successfully separates the fingerprint image region from the background image region and reduce the disturbance cause of the remainder and peeling image region at the same time, which the curves A, C, D can't accomplish. Hence, the curve B can be regarded as the boundary curve of this fingerprint image while the curve A, C and D can't be regarded as the boundary curve. So, the process of fingerprint image segmentation is to look for the boundary curve. If we can calculate the probability that a closed curve is the boundary curve of the fingerprint image according to the gray level of the fingerprint image, the closed curve with the biggest probability can be regarded as the boundary curve, for example, curve B as fig.1 showed. Obviously, such boundary curve probability density function (BCPDF) is required to satisfy the following conditions:

- (1) The value of BCPDF of the closed curve in background image region (e.g. curve A) is less than that of the boundary curve (e.g. Curve B).
- (2) The value of BCPDF of the closed curve within fingerprint image region (e.g. curve C) is less than that of the boundary curve (e.g. Curve B).



**Fig. 1.** Fingerprint image and the boundary curve

(3) The value of BCPDF of the closed curve that has crossed fingerprint image region and background image region (e.g. curve D) is less than that of the boundary curve (e.g. Curve B).

If we define inward ring and outward ring of a closed curve as fig.2 showed, compared with curves A, C, D, the boundary curve like curve B is required:

- (1) The outward ring of the boundary curve is in background image region exactly. (\\ denoted as outward ring as fig.2 showed).
- (2) The inward ring of the boundary curve is within fingerprint image region exactly. (/// denoted outward ring as fig.2 showed).

In this paper, we denote the outward background probability density function of a closed curve  $\Gamma$  as  $P_{out}(\Gamma)$  and the inward fingerprint probability density function as  $P_{in}(\Gamma)$ . Then, the value of  $P_{out}(\Gamma)$  is the probability that the outward ring is in the background region and the value of  $P_{in}(\Gamma)$  is the probability that the inward ring is in the fingerprint image region. So, if we denote BCPDF of  $\Gamma$  as  $PL(\Gamma)$ , then, we have:

$$PL(\Gamma) = P_{in}(\Gamma) P_{out}(\Gamma)$$

The following issue of fingerprint image segmentation is to find the closed curve  $\Gamma$  whose BCPDF value  $PL(\Gamma)$  is the biggest. And, the latter work is to calculate the outward background probability  $P_{out}(\Gamma)$  and the inward fingerprint probability  $P_{in}(\Gamma)$ .

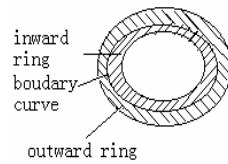


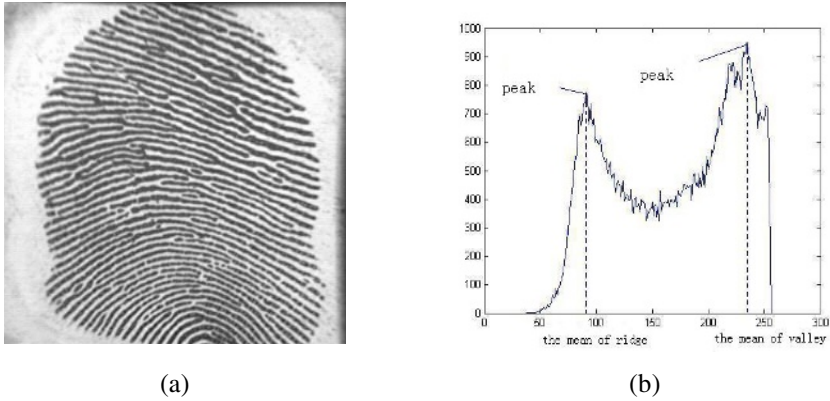
Fig. 2. Boundary curve and the inward ring

### 2.2 Calculation for the Outward Background Probability Density and the Inward Fingerprint Probability Density

Generally, fingerprint image can be segmented into two kinds of regions as background image region and fingerprint image region, which also can be labeled as ridge region and valley region. If we study the fingerprint image carefully, we can find that the gray levels of pixels in ridge region are very close, and so as valley region and background region. Moreover, gray levels of pixels in valley region or in background region are so close. Hence, the result is that the gray level will gather into two domains and there are two peaks in the corresponding histogram, as fig.3 showed. The gray level where pixels in background region or in valley region gathered is called as the mean of valley, so as the mean of ridge. Then, it can be considered that pixels in ridge region obey the normal distribution with the mean of ridge as the form:

$$p(i(x, y) | ridge) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(g_m - \mu_r)^2}{2\sigma^2}} \tag{1}$$

Where  $g_m$  denotes the gray level of the pixel  $i(x,y)$ ,  $\mu_r$  denotes the mean of ridge,  $\sigma^2$  denotes variance. And, pixels in background region or in valley region obey the normal distribution with the mean of valley as the form:



**Fig. 3.** The fingerprint image and the histogram, (a) is the original fingerprint image and (b) is the corresponding histogram

$$p(i(x, y) | valley) = p(i(x, y) | back) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(g_m - \mu_h)^2}{2\sigma^2}} \tag{2}$$

Where  $\mu_h$  denotes the mean of valley.

To a closed curve  $\Gamma$ , if the outward ring of  $\Gamma$  is in the background image region then every pixel is in the background image region and obeys the normal distribution with the mean of valley. In conclusion, the probability of the outward ring of  $\Gamma$  is in the background image region completely can be written as the form:

$$P_{out}(\Gamma) = \prod_{m=1}^k \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(g_m - \mu_h)^2}{2\sigma^2}} \tag{3}$$

Where k denotes the sum of pixels in the outward ring,  $g_m$  denotes gray level of the pixel  $i(x,y)$ ,  $\mu_h$  denotes the mean of valley.

But to the fingerprint image region, pixels in it are either in ridge region or in valley region. The gray distribution of valley region is the same to that of background region. Hence, according to whether the pixels are in ridge region or not, we judge the inward ring of  $\Gamma$  being in the fingerprint image region or not. We can see, the ridge line and the valley line are always appear by turns in the fingerprint image region. Hence, it can be considered that the sum of the pixels in ridge region is equal to that of pixels in valley region approximately. In other words, the sum of pixels in ridge region is equal to half of the sum of all pixels in fingerprint image region approximately. So, the inward fingerprint probability can be written as the form:

$$P_{in}(\Gamma) = (1 - \frac{1}{2}) \prod_{m=1}^k \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(g_m - \mu_r)^2}{2\sigma^2}} \tag{4}$$

Where N denotes the sum of all pixels in inward ring of  $\Gamma$ , k denotes the sum of pixels in ridge region,  $g_m$  denotes the gray level of the pixel in ridge region

and  $\mu_i$  denotes the mean of ridge. The left coefficient has guaranteed that the value of  $PL(\Gamma)$  is the biggest only if the sum of pixels in ridge region is half of the sum of all pixels in fingerprint image region, which is also the peculiarity of fingerprint image.

Now, we can calculate BCPDF  $PL(\Gamma)$  of any closed curve  $\Gamma$  in fingerprint image, through calculating the outward background probability  $Pout(\Gamma)$  and the inward fingerprint probability  $Pin(\Gamma)$ .

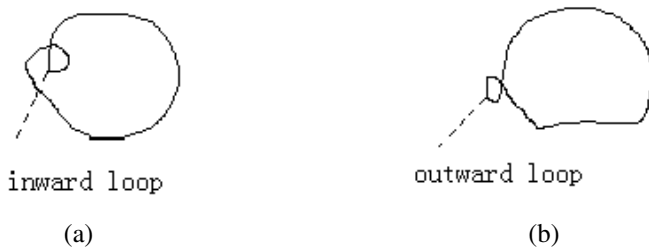
$$PL(\Gamma) = Pin(\Gamma)Pout(\Gamma) \tag{5}$$

In fingerprint image, the closed curve with the biggest value of BCPDF is the optimum solution of the fingerprint image segmentation. So, it is required to find the closed curve with the biggest value of BCPDF. A simple thought is looking for all closed curves in fingerprint image and finding one with the biggest BCPDF value. But, it is impossible to look for all the closed curves. Hence, there must be some approximate methods to do it. Markov Chain Monte Carlo (MCMC) will solve this kind of problem well. Generally, it required two steps with MCMC: (1) Generating Markov Chain according to the needs of problem. (2) Solving it with Monte Carlo and looking for the approximate answer.

### 2.3 Markov Chain of Boundary Curve in Fingerprint Image

Supposing the sequence  $\{\Gamma_1, \Gamma_2, \dots, \Gamma_n\}$  of the boundary curve in fingerprint image is a random Markov Chain, it can be known by the property of Markov Chain that  $P(\Gamma_{i+1} | \Gamma_i) = P(\Gamma_{i+1} | \Gamma_1, \Gamma_2, \dots, \Gamma_i)$ . In other words, the next state of the boundary curve  $\Gamma_{i+1}$  depends only on the current state  $\Gamma_i$ , but not the historical state  $\{\Gamma_1, \Gamma_2, \dots, \Gamma_{i-1}\}$ . There are two basic requirements for designing Markov chain dynamics. Firstly, it should be ergodic and aperiodic. Given any two closed curve  $\Gamma, \Gamma'$ , the Markov chain can travel from  $\Gamma$  to  $\Gamma'$  in finite steps. Secondly, it should observe the stationary equation.

This requirement is often replaced in a stronger condition: the so-called detailed balance equations. Brownian motion is a common stochastic process. So we design Markov chain in Brownian motion method as following:



**Fig. 4.** Two kinds of lonops that need to be gotten rid of: (a) is the inward loop and (b) is the outward loop

Step1: Supposing  $\Gamma_i$  is the set of the point  $x_i^k$ , every point  $x_i^k$  does Brownian motion,  $x_{i+1}^k = B(x_i^k)$ .

Step2: Connect points  $x_{i+1}^k$  in turn.  $\Gamma_{i+1}^0 = \{x_{i+1}^1, x_{i+1}^2, \dots, x_{i+1}^k, \dots, x_{i+1}^{n_i}\}$

Step3: Make up the collected curve and get rid of repeated loops, as the fig.4 showed,  $\Gamma_{i+1} = m(\Gamma_{i+1}^0)$ .

### 3 Markov Chain Monte Carlo and Genetic Algorithm

#### 3.1 Monte Carlo Simulation

To the curve Markov chain, as 2.3 discussed, we should introduce some optimize solutions to obtain the value of BCPDF in view of the calculation speed. Monte Carlo is such method of simulation that can converge at the boundary curve quickly. The key of Monte Carlo method is the selection of the kernel. Here we apply the Metropolis-Hastings scheme that has the following function:

$$P(\Gamma_{i+1} | \Gamma_i) = \min \left\{ 1, \frac{PL(\Gamma_{i+1})}{PL(\Gamma_i)} \right\}. \tag{6}$$

To any state  $\Gamma_i$  and the next state  $\Gamma_{i+1}$  of Markov chain, we calculate the shell and decide if transform or not based on the value of the transform probability  $P(\Gamma_{i+1} | \Gamma_i)$ . Now, we can summarize that we do it by MCMC method in the following steps.

Step1: Generate a potential Markov chain as 2.3 illustrated.

Step2: Supposing the current state is  $\Gamma_i$  and the next state is  $\Gamma_{i+1}$  in Markov chain as step1, calculate the shell  $P(\Gamma_{i+1} | \Gamma_i)$  as formula (6).

Step3: Generate a random variance  $u$  with uniform distribution at domain  $[0,1]$ .

Step4: If  $P(\Gamma_{i+1} | \Gamma_i) \geq u$ , Markov chain go to the next state  $\Gamma_{i+1}$  and go to step 2.

Step5: If  $P(\Gamma_{i+1} | \Gamma_i) < u$ , Markov chain refuse to transform,  $\Gamma_{i+1} = \Gamma_i$ , and go to step2.

Step6: If the number of continue repeated transforms is more than 50, stop the process and consider the current answer is the optimum answer.

#### 3.2 Genetic Algorithm

Because the Boundary Curve Markov Chain is generated by Brownian Motion, the next state may be very irregular. The price of convergent speed is big. The Genetic Algorithm will solve it. A genetic algorithm is a heuristically guided random search technique that concurrently evaluates thousands of postulated solutions. Biased random selection and mixing of the evaluated searches is then carried out in order to progress towards better solutions.

According to the Genetic Algorithm, the current closed boundary curve  $\Gamma_i$  can be divided into  $N$  divisions  $D_i^k, k = 1, 2, \dots, N$  and code it with number "0", correspondingly, the next  $N$  divisions  $D_{i+1}^k$  is coded with "1". If we randomly select code "0" or "1" to combine the codes (d,d,...,d), d is "0" or "1", the probability will be  $2^N$ . In other words, the probability of the new connected curve  $\Gamma_{i+1}^j$  is  $2^N$ . Then, from the  $2^N$  curves, we randomly select  $M$  curves  $\Gamma_{i+1}^{j_k}, k = 1, 2, \dots, M$  and calculate the BCPDF  $PL(\Gamma_{i+1}^{j_k})$ , lastly, we select the curve  $\Gamma_{i+1}^{j_k}$  which has the biggest BCPDF value as the next Markov state  $\Gamma_{i+1}$ . The steps of MCMC&GA is,

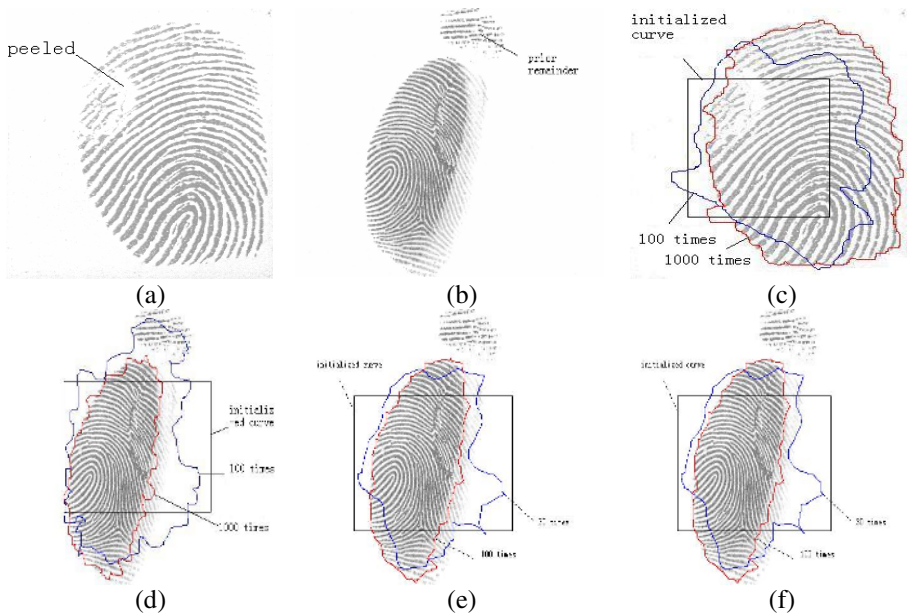
Step1: Generate a potential Markov Chain as 2.3 illustrated.

Step2: optimize the Markov Chain with Genetic Algorithm, as 3.2 showed.

Step3: Monte Carlo simulates the optimized Markov Chain to the convergent solution  $\Gamma$ , as 3.1 showed.

### 4 Experiment Results and Conclusion

To examine the effect of the algorithm proposed in the paper, we chose DB\_B in the public data-BVC2004, which is considered the most hard fingerprint image database to segmented in common and disposed it with MCMC and MCMC&GA respectively.



**Fig. 5.** Some fingerprint images and the segmentation results by MCMC and MCMC&GA ((a) and (b) are the original low-quality fingerprint images; (c) and (d) are the corresponding segmentation results of MCMC to (a) and (b) respectively; (e) and (f) are the corresponding segmentation results of MCMC&GA to (a) and (b) respectively).

Fig.5 is the representative fingerprint images and the fingerprint image segmented results. The (a) and (b) are the original fingerprint images with strong noise, the (c) and (d) are the segmentation results with MCMC when allowed to run for 100 iterations and 1000 iterations, the (e) and (f) are the results with MCMC&GA for 30 iterations and 100 iterations. The experimental results indicate that MCMC&GA is more effective.

The paper proposes the method of fingerprint image segmentation based on MCMC&GA. It takes the closed curve in fingerprint image as research object, randomly generates Markov chain of closed curves, then, it is simulated by Monte Carlo method to convergent to the boundary curve whose boundary curve probability function is the biggest.

The primary experimental results indicate that MCMC&GA method is robust to the fingerprint image with stronger disturbance of background, especially to the peeled fingerprint and the prior remainder fingerprint.

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