

Fingerprint Image Segmentation Based on Quadric Surface Model*

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Abstract. It is essential to segment fingerprint image from background effectively, which could improve image processing speed and fingerprint recognition accuracy. This paper proposes a novel fingerprint segmentation method at pixel level based on quadric surface model. Three parameters, *Coherence*, *Mean* and *Variance* of each pixel are extracted and spatial distribution model of fingerprint pixels is acquired and analyzed. Our study indicates that the performance of fingerprint image segmentation with a linear classifier is very limited. To deal with this problem, we develop a quadric surface formula for fingerprint image segmentation and acquire coefficients of the quadric surface formula using BP neural network trained on sample images. In order to evaluate the performance of our proposed method in comparison to linear classifiers, experiments are performed on public database “FVC2000 DB2”. Experimental result indicates that the proposed model can reduce pixel misclassification rate to 0.53%, which is significantly better than the linear classifier’s misclassification rate of 6.8%.

1 Introduction

In recent years, automatic fingerprint identification has always been a research focus in academic research and industry field of the world. With the efforts of many researchers, the main technical system of automatic fingerprint identification has been built already, and it has been applied in many fields. However, to meet the needs of applications, it’s necessary to improve the performance of key algorithms, such as fingerprint image preprocessing, feature extraction and fingerprint matching algorithms, etc. We can say that the researches on fingerprint identification algorithms have developed into a procedure full of competitive.

Quality of fingerprint itself and image capture conditions directly influence the performance of the system, which makes fingerprint preprocessing be a necessary step. In addition, the separation of fingerprint image from background is the first step in preprocessing. A valid and effective segmentation could not only reduce time consumed on image preprocessing and minutiae extraction time but also improve the reliability of identification. So research on fingerprint segmentation has important meaning to the whole fingerprint identification system.

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There are many literatures about fingerprint image segmentation. Overall, the current approaches for fingerprint image segmentation can be classified into the following two categories. One method is based on the block level. B.M.Methre classifies each block into foreground or background according to the distribution of the gradients and gray-scale variance in that block [1,2]. X.Chen et al use linear classifier to classify the block [3]. L.R.Tang et al propose a fingerprint segmentation method based on D-S evidence theory [4]. It is carried out by the direction and contrast of block. S.Wang et al make use of contrast and main energy ratio to complete the separation of valid fingerprint part from background [5]. Q.Ren et al give the method based on feature statistics [6]. Obviously, the resolution of this method only reaches the block level, and the border of foreground and background obtained by this method is rather serrate, which makes it difficult to judge the reliability of features extracted from the border. The other method is based on pixel level. A.M.Bazen [7,8] et al use three features of pixel and establish a linear classifier to implement a pixel-based segmentation. Up to the present, prevailing method of fingerprint segmentation is still based on block. The number of method based on pixel is very little.

In this paper, a method based on quadric surface model for fingerprint image segmentation is presented. According to the spatial distributions of the pixel features in the foreground and background areas of database, it implements the segmentation of fingerprints using quadric surface model. In algorithm, we transfer the quadric discriminant to broad sense discriminant for reducing the computation complexity. Experiments have shown that the performance is excellent.

This paper is organized as follows. Section 2 studies the spatial distribution of pixels based on CMV. Section 3 presents development of the quadric surface model. Section 4 is the experimental results on fingerprint database and section 5 gives conclusions and some discussions about this presented method.

2 Spatial Distribution of Pixels Based on CMV

This section consists of two parts: one is description of pixel features and the other is spatial distribution of pixels based on above features.

2.1 Description of the Pixel Features

In brief, segmentation is the classification of pixels, so the first important problem of the fingerprint segmentation is to define proper parameters to describe the pixel features. The valid and distinguishable features of the pixel should play vital roles in the segmentation. For the pixels in the valid fingerprint foreground and background, the parameters should sufficiently represent their differences on these features. In this paper, *Coherence*, *Mean* and *Variance* are selected as the pixel features [8]. The definition of each feature is as following.

2.1.1 Coherence(C). The *Coherence* of the pixel measures how well the gradients are pointing in the same direction around a pixel, which is abbreviated as *Coh* [7,8,9]. Since a fingerprint mainly consists of parallel line structures, the coherence will be considerably higher in the foreground than in the background. In a window W around a pixel, it is defined as follows:

$$Coh = \frac{\left| \sum_W (G_{s,x}, G_{s,y}) \right|}{\sum_W \left| (G_{s,x}, G_{s,y}) \right|} = \frac{\sqrt{(G_{xx} - G_{yy})^2 + 4G_{xy}^2}}{G_{xx} + G_{yy}} \tag{1}$$

Where $(G_{s,x}, G_{s,y})$ is the squared gradient, $G_{xx} = \sum_W G_x^2$, $G_{yy} = \sum_W G_y^2$, $G_{xy} = \sum_W G_x G_y$ and (G_x, G_y) is the local gradient. W is the two-dimensional low-pass template sized of 17×17 used for noise reduction. From the formula, we can see that the coherence is among $[0,1]$. The coherence of pixel in foreground is near 1, while in background it is close to 0, because the directions of noise areas are in chaos.

2.1.2 Mean (M). The second pixel feature is the average gray value. It measures how gray the pixel is. For most fingerprint sensors, the ridge-valley structures can be approximated as black and white lines. So the foreground is composed of black and white lines, while the background, where the finger doesn't touch the sensor, is rather white. This means that the mean gray value in the foreground is lower than it is in the background. The local mean of the pixel is defined as follows:

$$Mean = \sum_W I \tag{2}$$

Where I is the local intensity of the image. The definition of W is the same as above.

2.1.3 Variance (V). The *Variance* is the third pixel features used in the algorithm, which measures the gray variance around the local area. In general, the variance of the ridge-valley structures in the foreground is higher than the variance of the noise in the background. It is defined as follows:

$$Var = \sum_W (I - Mean)^2 \tag{3}$$

Where the definitions of I, W are the same as above too.

2.1.4 CMV Normalization. Since the value ranges of the three features are different, after extracting the three features of a pixel, we need to normalize them to the range of $[0,1]$.

2.2 Spatial Distribution of Pixels Based on CMV

It is important to make deep insight into the spatial distribution based on the CMV of the pixel. According to this distribution, we may get the appropriate segmentation model by certain technique and achieve the satisfactory segmentation results.

Therefore, 60 representative fingerprint images from FVC2000 Database 1 have been selected as samples. Each pixel's CMV values of all the images are extracted and the spatial distributions of these pixels are shown in Fig. 1(a)(where the yellow section is composed of the pixels in the background, while the blue sections is composed of the pixels in the foreground. Which parts the pixels belongs to is decided

manually and the three coordinates of a point in the figure represented the CMV values of a pixel in the fingerprint image). Fig. 1(b) is obtained by extending the foreground and background pixels in Fig. 1(a) in order to see the overlapping section clearly.

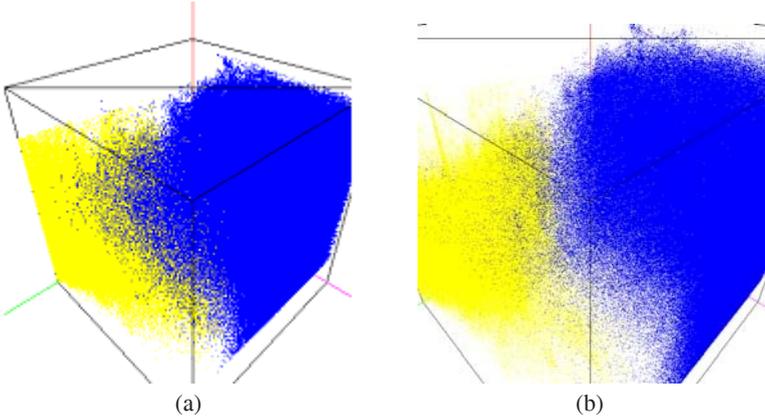


Fig. 1. Spatial distributions of pixels based on CMV (a)(the yellow section is composed of the pixels in the background, while the blue section is composed of the pixels in the foreground), (b)(obtained by extending (a))

From the distributions of pixels, we can see that the pixels of different parts (background and foreground) have different clusters, which indicates that the pixels of the same class have similar features, and that two clusters are not apart completely.

Obviously, based on this spatial distribution, it is difficult to get satisfactory results only using the plane. In addition, linear classifier has its limitations and can't solve such nonlinear problems. All these factors make us select the quadric surface model.

3 Development of Quadric Surface Model

3.1 Selection of Segmentation Model

As we can see from the spatial distribution based on CMV of pixels, pixels from foreground and background are hard to separate with plane. if we select the quadric surface model, we may get better results. In general, suppose the quadric surface model is as follows:

$$g(x) = x^T Wx + w^T x + w_0 = \sum_{k=1}^d w_{kk} x_k^2 + 2 \sum_{j=1}^{d-1} \sum_{k=j+1}^d w_{jk} x_j x_k + \sum_{j=1}^d w_j x_k + w_0 \quad (4)$$

W is a real symmetric matrix and w is a d -dimension vector. From (4), $L = \frac{1}{2}d(d+3)+1$ coefficients are needed and the calculation cost is large. So in order to simplify the algorithm, we define W as a matrix that only elements on main

diagonal is non-zero, others are all zero. $x = [C, M, V]^T$, and $d = 3$. Then formula (4) can be rewritten as following,

$$g(x) = x^T Wx + w^T x + w_0 = \sum_{k=1}^3 w_{kk} x_k^2 + \sum_{j=1}^3 w_j x_k + w_0. \tag{5}$$

Apparently, this quadric discriminant is nonlinear, which is difficult to handle with. So by increasing the dimension, the above can be translated into a broad sense linear discriminant. Under this theory, formula (5) can be translated to following,

$$\begin{aligned} g(x) &= x^T Wx + w^T x + w_0 = \sum_{k=1}^3 w_{kk} x_k^2 + \sum_{j=1}^3 w_j x_k + w_0 \\ &= \sum_{i=0}^6 a_i y_i = a^T y \end{aligned} \tag{6}$$

Where,

$$\begin{aligned} y &= [y_0, y_1, y_2, y_3, y_4, y_5, y_6]^T = [C^2, M^2, V^2, C, M, V, 1]^T \\ a &= [a_0, a_1, a_2, a_3, a_4, a_5, a_6]^T \end{aligned}$$

Then the rest problem is how to get the coefficients a_i ($0 \leq i \leq 6$) from a lot of samples.

3.2 Acquisition of Relative Coefficients

BP network algorithm is a relatively good feed-forward neural network algorithm in theory, and it is also widely used in practice. It is a supervised leaning algorithm used in many fields to simulate some very complicated nonlinear relationships when the conventional methods do not work. In this paper, we use the perception of BP learning algorithm to obtain the relative coefficients a_i ($0 \leq i \leq 6$).

Different fingerprint database need to establish different model coefficients [8], in this paper, 30 representative fingerprint images respectively from FVC2000 DB1, FVC2000 DB2 and FVC2000 DB3 are selected to get according model coefficients.

This neural network has 7 inputs and 1 output.

Inputs of the network: $y = [y_0, y_1, y_2, y_3, y_4, y_5, y_6]^T$,

Weight vector: $a = [a_0, a_1, a_2, a_3, a_4, a_5, a_6]^T$.

Transfer function:

$$f(g) = \frac{1}{1 + e^{-g}}. \tag{7}$$

The structure of the network is shown in Fig. 2, where the real directional arrow denotes the direction of working signals, while the dashed directional arrow denotes the back error signals.

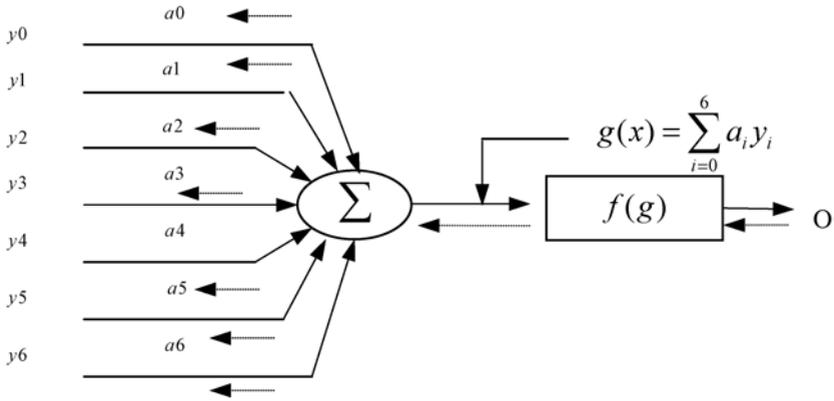


Fig. 2. The structure of network

The segmentation method presented in this paper uses supervised training. The classifications of the pixels (background or foreground) that are involved in the network training are selected manually.

The fingerprint image samples are the same as described above, which are representative enough.

3.3 Implementation of Segmentation

Since we have got the coefficients of the models, segmentation can be implemented using quadric surface model. Suppose $\hat{\omega}_0$ denotes the pixel that belongs to foreground, $\hat{\omega}_1$ denotes the pixel that belongs to background and $\hat{\omega}$ denotes the pixel's classification. The decision function used in the segmentation is:

$$\hat{\omega} = \begin{cases} \hat{\omega}_0 & \text{if } g > 0 \\ \hat{\omega}_1 & \text{if } g \leq 0 \end{cases} \quad (8)$$

Where the definition of g is in formula (6).

3.4 Visualization of Quadric Surface Model

Using the above neural network model to train three different sample databases, we obtain three different sets of coefficients, indicating that the optimal quadric surface models for fingerprint images acquired by different sensors are different. It also proves the theory proposed by A. M. Bazen regarding the necessity of training different fingerprint databases independently. The quadric surface models obtained from fingerprint training samples in database FVC2000 DB1, DB2 and DB3 are illustrated in Fig.3 (a), (b) and (c), respectively. Blue dots stand for foreground pixels, yellow dots represent background, and red surfaces denote segmentation surfaces.

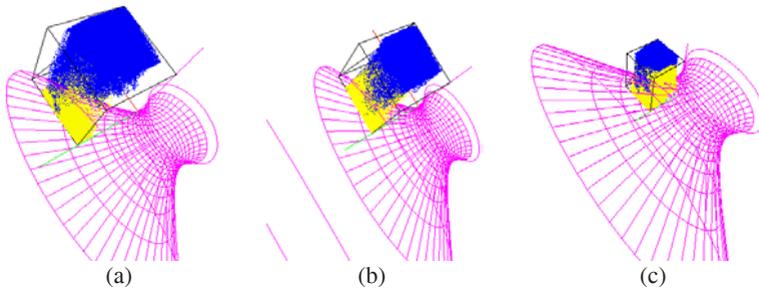


Fig. 3. The quadric surface models (a)(obtained from FVC2000 DB1), (b)(obtained from FVC2000 DB2), (c)(obtained from FVC2000 DB3)

4 Experimental Results

Experiment 1: Evaluating the validity of the model for each fingerprint database. From each database (FVC DB1, DB2, DB3), select 300 images respectively and form testing sets (TestDB1, TestDB1, TestDB3) different from training sets. Now we are intended to evaluate if the model is validate for the training databases.

For each pixel, calculate CMV values and put them to formula (6), then calculate (7) to get $f(g)$, and make it among $[0,1]$. The probability density of Pixels from foreground and background of each database is given in Fig. 4. Make $(0,1)$ to $(0,10)$, and disperse it to 10 equal parts.

Define $Fingerf(x) x \in \{0,1,2,3,4,5,6,7,8,9\}$ as the probability of foreground pixels in $(x,x+1)$ and $Fingerb(x) x \in \{0,1,2,3,4,5,6,7,8,9\}$ as the probability of background pixels in $(x,x+1)$.

Obviously, $\sum_{x=0}^9 Fingerf(x) = 1$ and $\sum_{x=0}^9 Fingerb(x) = 1$. The point $x = 5$ is the threshold of differentiating pixels from foreground and background.

Experiment 2: Doing comparison experiments with A.M.Bazen’s method on the second database of FVC2000. The parameters that measure the performance of the segmentation are defined as follows:

$p(\hat{\omega}_1 | \omega_0)$: The probability that a foreground pixel is classified as background.

$p(\hat{\omega}_0 | \omega_1)$: The probability that a background pixel is classified as foreground.

\bar{p}_{error} : The average of $p(\hat{\omega}_1 | \omega_0)$ and $p(\hat{\omega}_0 | \omega_1)$.

The experimental results for the Database 2 of FVC2000, using our method and the method presented in [8], are shown in Table 1.

Table 1. The segmentation results of our method and the method in[8] on FVC2000 DB2

Method	$p(\hat{\omega}_1 \omega_0)$	$p(\hat{\omega}_0 \omega_1)$	\bar{p}_{error}
Our method	0.21%	0.85%	0.53%
Method in [6]	6.2%	7.4%	6.8%

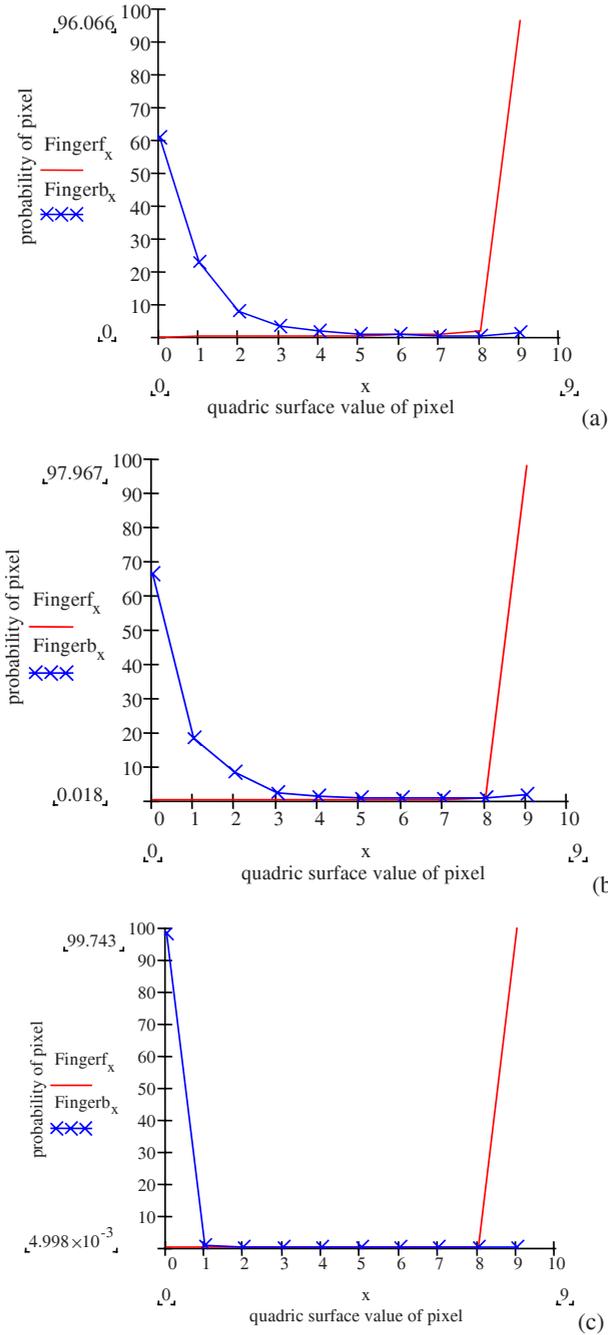


Fig. 4. The probability of pixels from foreground and background in (a)(TestDB1), (b)(TestDB2) and (c)(TestDB3)

5 Conclusion and Discussions

In this paper, a fingerprint segmentation algorithm using quadric surface model is proposed, which is based on the spatial distribution of the pixels derived from coherence, local mean and local variance. The spatial distribution of the pixels has special meaning for the segmentation. Experimental results on fingerprint images of FCV2000 database have demonstrated that the proposed segmentation method has excellent performance and the segmentation result is significantly better than that of the linear classifier.

The selection of curve surface model, and finding the optimal model to achieve more satisfactory segmentation result would be the next important step in fingerprint image segmentation study. Moreover, how to prove that the distribution of pixels based on CMV from foreground and background is a non-linear problem in theory and the reasonable feature selection of pixel are another worthwhile research.

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