Singular points detection based on multi-resolution in fingerprint images

Dawei Weng, Yilong Yin*, Dong Yang

School of Computer Science and Technology, Shandong University, Jinan 250101, PR China

1. Introduction

A fingerprint is an oriented texture pattern of ridges and valleys possessing a limited range of spatial frequencies. One of the most interesting characteristics of fingerprint patterns is that their global structure is dominated by a small number of singular points, namely core and delta, which are viewed as global features of fingerprints. A core point is defined as a concentrate region where the ridges' curvature is converging to a local maximum [1], while a delta point is defined as the confluent point of three different flowlike directions [2] (see Fig. 1). Singular points have been traditionally used in fingerprint classification [3–5] to reduce search space in large database and have been used as reference points for fingerprint alignment [6].

There are several published studies on singular points detection in fingerprint images. Srinivasan and Murthy [1] proposed an algorithm using local orientation histogram based on block directional images to detect singular points. The place where the local orientation histogram does not exhibit a prominent peak is likely to be a singularity. Koo and Kot [10] presented a multi-resolution approach to determine singularities. At each resolution, they derive, from the orientation image, a curvature image whose blocks indicate the local degree of ridge curvature, and the high-curvature regions at finer resolutions are denoted as singular regions. Nilsson and Bigun [11] used two complex filters tuned to detect singular points from the orientation field, which is based on multi-resolution analysis. The points where the magnitude of complex filter response is high are retained as singular points. Chikkerur and Ratha [12] improved Nilsson's method by using three additional certainty maps that represent heuristics about the likely positions and orientations of cores and deltas. Poincaré index, the most classical way to detect singular points, was proposed by Kawagoe and Tojo [13], and Bazen and Gerez [2] provided an alternative implementation of the Poincaré index method, based on Green's theorem. However, methods based on Poincaré index are prone to be affected by noise because only local information is utilized, which is not enough to discriminate true singular points from spurious detections. There are also many works combining local and global information to detect singularities. Zhou et al. [14] addressed singular points detection based on DORIC feature and global constraints, and DORIC is a...
novel feature proposed in his paper, namely, Differences of the ORientation values along a Circle. Fan et al. [15] combined Zero-pole Model and Hough Transform (HT) to detect singular points. Zero-pole Model, first proposed by Sherlock and Monro, reveals that the orientation field of fingerprint is dominated by the number and positions of singular points. In [15], Zero-pole Model is employed to detect singularities by determining the parameters of this model, and it is proved that Zero-pole Model is practical and ideal to describe the topology of fingerprint image. Jin et al. [16] extracted singular points from multi-scale Gaussian filtered orientation field to achieve pixel-level accuracy. Besides these methods, there are still neural networks approach [17], Markov Model approach [18], FOMFE-Based method [19], inconsistency feature-based method [20] and so on. Among all the singular points detection algorithms, most of them, although using various techniques, critically relied on orientation fields of fingerprint images. However, orientation field computation itself is a tough work, especially when dealing with poor quality fingerprint images. Furthermore, the choice of block-wised or pixel-wised orientation field is a difficult trade-off as well: block-wised direction has strong capability to avoid spurious detections, but lower precision in the location of singular points, and pixel or relatively smaller block direction can locate singular point more precisely, but cannot effectively counteract the noises impairments resulting in spurious detection.

In this study, to detect singularities accurately and reliably, we suggest a model, which combines local information and global information while reducing the dependence on single orientation field. Based on an analysis of model-based and gradient-based orientation fields using Zero-pole Model and the least mean square orientation estimation algorithm, respectively, we ameliorate the traditional Poincaré index method, making it appropriate to capture the orientation changes along a close circle around the candidate singularities in different resolution orientation fields. The multi-resolution characteristic of fingerprint pattern is explicitly reflected in its orientation fields. Orientation field estimated by a small block size exhibits the orientation changes of the innermost ridge in a singular point region, while using relatively bigger block size can reflect orientation changes of relatively outside ridges. Orientation fields based on the same block size but different block position placements discretely exhibit different aspects of the continuous fingerprint pattern, maybe only some of which contain the Poincaré index features for a specific fingerprint singularity. Relationships of singularities detected in different resolution orientation fields by improved Poincaré index can be used to properly remove noises-introduced singular points, and singularities in high resolution orientation field can be determined as the true points which are located more precisely than in low resolution orientation field. Corresponding to one true singularity, many relative singular points are located in these orientation fields, and singularities detected in orientation fields based on the same block size but different block positions, which can be viewed as a type of tessellation for fingerprint with square grids and it is a number of shifts along x-axis and y-axis directions with certain interval, intensively concentrate in a small area. This model incorporates these information into singular points detection, making our algorithm more robust and accurate than methods that use single information only. Furthermore, as a framework, this model may be directly exploited in techniques relying on orientation field of fingerprints. The methods of [10,11,21] are also based on multi-resolution analysis, but the detailed implementation manner of them are totally different. Algorithms [10,11] use curvature image or complex filters to realize multi-resolution analysis. However, our algorithm presents a framework of multi-resolution orientation fields, based on wavelet fundamental functions and Sampling theorem.

We corroborate our technique by experiments on a state-of-the-art database NIST-4 and public fingerprint databases FVC02 DB1, DB2.

2. Analysis for orientation field of fingerprint

Zero-pole Model was proposed by Sherlock and Monro [7] to model orientation field of fingerprints. The orientation o(z) of a point z is an argument of the complex function p(z) defined by using the following equations:

$$p(z) = \sqrt{e^{i\phi_c} \left(\sum_{i=1}^{k} \arg(z-z_{ci}) - \sum_{j=1}^{l} \arg(z-z_{dj})\right)}$$  \hspace{1cm} (1)

$$o(z) = (\arg(p(z))) \mod \pi$$  \hspace{1cm} (2)

where z_{ci} and z_{dj} are the i-th core and the j-th delta of fingerprint, and o_c is a constant correction term. In order to observe conveniently, we transformed (2) to another equivalent form as below:

$$o(z) = \left[ o_c + \frac{1}{2} \times \left( \sum_{i=1}^{k} \arg(z-z_{ci}) - \sum_{j=1}^{l} \arg(z-z_{dj}) \right) \right] \mod \pi$$  \hspace{1cm} (3)

As (3) indicates, orientation at a point z is the sum of influence of all cores and deltas. In reality, each singularity has a mainly dominated area that orientation in this area is mainly determined by its closest singular point. Based on this analysis, (3) can be simplified as follows:

$$o(z) = [o_c + \frac{1}{2} \times \arg(z-z_i)] \mod \pi$$  \hspace{1cm} (4)

or

$$o(z) = [o_c + \frac{1}{2} \times \arg(z-z_d)] \mod \pi$$  \hspace{1cm} (5)

and they represent the case of one core or one delta, respectively. Zero-pole Model is an ideal orientation model to describe the topology of fingerprint images. In contrast to traditional application, Fan et al. [15] employ Zero-pole Model in an inverse fashion to detect singular points by determining the parameters of the Zero-pole Model. In [15], orientation of singular points is defined based on Zero-pole Model. From the orientation definition of delta, we can get that the orientation of delta has three values, which can well explain why delta is a confluent point of three different flow direction. Besides, Zero-pole Model had been used to reconstruct a noisy image [22]. All the above further prove the validity of Zero-pole Model to model the orientation field of fingerprints. In [15], the orientation of the m-th core e_m is defined as follows:

$$o(z_{cm}) = \left[ \frac{1}{2} \times \left( \sum_{i=1}^{k} \arg(z_{cm}-z_{ci}) - \sum_{j=1}^{l} \arg(z_{cm}-z_{dj}) \right) + o_c + \frac{1}{2} \times o(z_{cm}) \right] \mod \pi$$  \hspace{1cm} (6)
Fan et al. [15] also derived the equal form of (6) as described below:

\[
o(z_{cm}) = \left[2 \times o_{\infty} + \left( \sum_{i=1, i \neq m}^{k} \arg(z_{cm}-z_i) - \sum_{j=1}^{l} \arg(z_{cm}-z_q) \right) \right] \mod \pi
\]

(7)

Considering the mainly dominated region of each singular point, we simplify (6) and (7) as follows:

\[
o(z_{cm}) = \left[ \frac{1}{2} \times o(z_{cm}) + o_{\infty} \right] \mod \pi
\]

(8)

and

\[
o(z_{cm}) = \left[ 2 \times o_{\infty} \right] \mod \pi
\]

(9)

However, (6) and (7) are not always equal as well as (8) and (9), because the definition of (6) has an implicit constraint that the value of \(o_{\infty}\) is restricted in a limited range. Eq. (8) is equal to

\[
\frac{1}{2} \times o(z_{cm}) + o_{\infty} = k\pi + o(z_{cm}), \quad k \in \mathbb{Z}
\]

(10)

where \(Z\) denotes integer. From (10), it can be seen that \(o_{\infty}\) has a special range depending on the range of fingerprints orientation field. Usually, orientation of fingerprint is in the range \((0, \pi)\), namely, the range of \(o(z_{cm})\) is \([0, \pi)\), so the range of \(o_{\infty}\) is

\[
k\pi \leq o_{\infty} < \frac{1}{2} \times \pi + k\pi, \quad k \in \mathbb{Z}
\]

(11)

The existence of (11) implies that (6) may not be suitable for some cases that possess an \(o_{\infty}\) which does not belong to (11). In order to handle this problem, we present another definition based on the observation for a great many actual fingerprints and the comprehension for multi-resolution characteristics of fingerprint ridge pattern. The multi-resolution property of fingerprint is most conspicuously reflected on orientation field around core points. Core point has the same direction with the points on the line orthogonal to the direction of core point, as shown in Fig. 2. All the properties of these points involving core point are the same, except for resolution. As analyzed above, we redefine the direction of core point as follows:

\[
o(z_{cm}) = \left[ \frac{1}{2} \times \left( o(z_{cm}) \pm \frac{\pi}{2} \right) + o_{\infty} \right] \mod \pi
\]

(12)

and generalize it

\[
o(z_{cm}) = \left[ \frac{1}{2} \times \left( \sum_{i=1, i \neq m}^{k} \arg(z_{cm}-z_i) - \sum_{j=1}^{l} \arg(z_{cm}-z_q) \right) \right]
\]

\[
+ \frac{1}{2} \times \left( o(z_{cm}) \pm \frac{\pi}{2} \right) + o_{\infty} \mod \pi
\]

(13)

where \(o(z_{cm}) \pm \pi/2\) indicates an orthogonal direction oriented from high curvature point to low curvature point (see Fig. 2). Eq. (13) can handle the previous problem efficiently, and the cues that these points are nearly linear can be incorporated into the detection of core points as a relatively global feature.

Through further analysis for the orientation field generated by Zero-pole Model, we obtain another interesting conclusion that the orientations of pixels on a closed curve around core point have a sudden change from \(\pi_{-}\) to \(\pi_{+}\) along counterclockwise direction where \(\pi_{-}\) denotes the value that infinitely approximate \(\pi\) from the negative direction of real number axis while \(\pi_{+}\) from the positive direction. For a delta point, the transition from \(\pi_{-}\) to \(\pi_{+}\) occurs along clockwise direction. Assume that a point labeled as \(z_i\) possesses direction \(\pi_{-}\) then (4) is equal to

\[
o(z_i) = \left[ o_{\infty} + \frac{1}{2} \times \arg(z_i-z_c) \right] \mod \pi
\]

(14)

Furthermore,

\[
\arg(z_i-z_c) = 2k\pi + 2 \times \pi_{-} - 2 \times o_{\infty}, \quad k \in \mathbb{Z}
\]

(15)

Being a continuous flow pattern, the point, which is infinitely close to \(z_i\) and meets

\[
\arg(z_i-z_c) = 2k\pi + 2 \times \pi_{+} - 2 \times o_{\infty}, \quad k \in \mathbb{Z}
\]

(16)

must exist in fingerprint. So the direction of this point labeled as \(z_j\) is \(0_{\pm}\), and where forms a sudden change of direction. Fig. 3 shows the orientation field modeled by Zero-pole and the direction change on it. Structurally speaking, the sudden change of orientation is indicative of the distinctive shape of ridges around core point. On the other hand, from the view of computation methods of orientation field, gradient-based [13] and model-based orientation fields differ significantly in their orientation change, caused by the variation of block size allowed in gradient-based algorithm. And the gradient-based orientation field computed by using small block size can just retain the sudden orientation change of high curvature point, e.g. \(w_3\) shown in Figs. 2 and 4, but cannot embody the sudden orientation change of low curvature region just providing gradual variation of orientation field, while big block size can retain the sudden orientation change of low curvature point, e.g. \(w_3\) shown in Figs. 2 and 4, but smooth the orientation change of high curvature region. In other words, model-based orientation field can be viewed as an ideal case, and gradient-based orientation fields

![Fig. 2. Singular point (w3), placement of similar points with different curvature (w1, w2, w3), and orientation fields of these regions. (a) Left loop, (b) right loop, and (c) double loop. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)](image)

![Fig. 3. Orientation field generated by Zero-pole Model, and mark of orientation change superimposed on it: (a) a core and (b) a delta.](image)
direction. From the above formulae, it can be seen that the Poincaré index value, calculated using (18), is not exactly equal to 1/2, but changes in a narrow neighborhood of 1/2,[1/2−δ,1/2+δ] corresponding to different ridges curvatures. Similarly, the calculated Poincaré index value of delta point changes in a range of [−1/2−δ,−1/2+δ]. Ridge curvatures of singular region are significantly different for a wide variety of fingerprints, so it is difficult to properly determine the value of δ. If selecting a larger δ, more spurious points will be detected; if δ is too small, it may lead to missing real singularities.

Furthermore, in noisy region, when calculating the sum of orientation differences, the last two conditions may be used more than once because of the randomness of orientation field caused by various noises. Hence, some noisy points may be detected as true singularities because their Poincaré index values occasionally satisfy requirements under this condition. However, orientation field around genuine singularity should change continuously and would only satisfy one of the last two conditions of (18) once. In a way, this results in the poor anti-noise performance of traditional Poincaré index.

Finally, when detecting delta points, the case, satisfying condition π−δ(k) of (18) for genuine delta points, only occurs once for Poincaré index method, so that the value of orientation difference δ(k) of that time should be π, and only under this condition, the sum of orientation differences on the closed curve π−2δ(k) is equal to −π required by true delta points. However, this requirement is rather rigid as that difference heavily depends on ridges curvature of singular region which is significantly different for various fingerprints. We think this is another great flaw to result in missing genuine delta points.

3. The improved Poincaré index

3.1. Analysis for traditional Poincaré index

Currently, Poincaré index-based methods are the most popular algorithms for singular points detection. Poincaré index was first used to extract singular points by Kawagoe and Tojo [13] in 1984. Let \( C \) denote a vector field and \( C \) denote a curve immersed in \( G \), then the Poincaré index \( \text{Poin}C \) is defined as the total rotation of the vectors on curve \( C \). However, orientation field of fingerprints is not a pure vector field as its elements are unoriented directions, usually defined in a range of \([0,\pi)\) or \([−\pi/2,\pi/2)\), not \([0,2\pi)\). So the Poincaré index value of pixel \((i,j)\) is usually computed as follows [13]:

\[
\text{Poin}(i,j) = \frac{1}{2\pi} \sum_{k=0}^{N-1} \Delta(k)
\]

\[
\Delta(k) = \begin{cases} 
\delta(k) & \text{if } |\delta(k)| < \frac{\pi}{2} \\
\pi + \delta(k) & \text{if } \delta(k) \leq \frac{\pi}{2} \\
\pi - \delta(k) & \text{otherwise}
\end{cases}
\]

\[
\delta(k) = O(\psi_x(k'),\psi_y(k')) - O(\psi_x(k),\psi_y(k))
\]

\[
k' = (k+1) \mod N_p
\]

where \( \psi_x(k) \) and \( \psi_y(k) \) are the x and y coordinates of the kth point on the closed curve which is centered at the given point \((i,j)\) and composed of \(N_p\) pixels, \( O \) denotes a fingerprint orientation field. If the Poincaré index value is 1/2, then define the given point \((i,j)\) as a core point; if the value is −1/2, then the point is defined as a delta point.

Each difference is adjusted in terms of different conditions of Formula (18) when summing the orientation differences between adjacent elements of the closed curve along counterclockwise using different block sizes embody different properties of ideal orientation field, respectively. For delta point, because of lacking multi-resolution characteristic, the sudden orientation change around delta point is just a reflection of local ridges structure.

3.2. Improved Poincaré index method

Traditional Poincaré index is very precise and relatively simple to compute, but easy to be affected by noise, so it is desirable to obtain an improved one which is both accurate and consistent. Many previous works have been proposed to improve the traditional Poincaré index method. In contrast to the traditional Poincaré index, Bazen and Gerez [2] compute the rotation of the orientation image and then perform a local integration in a small neighborhood of each element, instead of summing angle differences along a closed circle. Zhang and Yan [5] determine singularities’ types and locations according to the size and type of each cluster obtained by clustering the singular points detected by the Poincaré index. In order to promote anti-noise capability of traditional Poincaré index, Wei et al. [23] sum the absolute value of orientation differences on a closed circle to detect singular points.

![Fig. 4. Diagram of different block sizes: (a) original image, (b) small block, (c) medium block, (d) big block. The red grid indicates the optimal block position to capture the transition point of orientation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)](image)

![Fig. 5. Schematic diagram of orientation variation around singularity: (a) a core and (b) a delta. The red circle labels the points where a sudden orientation change occurs. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)](image)
From Section 2, it has been shown that there is an orientation change from $\pi$ to 0 around core point along counterclockwise direction, while along clockwise direction for delta point. This rule is also applicable when defining the orientation field in the range $[-\pi/2, +\pi/2]$. From the local distinctive ridge pattern around singular point as exhibited in Fig. 5, we summarize three important properties as follows.

(1) The variation of orientation angles computed by gradient-based method [24–26] on the closed curve is continuous around singular points. The closed curve is composed of two types of pixels, the first type has positive directions while the second type has negative directions. Furthermore, the direction transitions from the first type to the second type and from the second type to the first type occur only once, respectively.

(2) These two transition points also explained in Section 2 are mutually different. One with the smaller orientation difference, namely absolute $\delta(k)$, occurs at the points whose orientations are near to the angle of 0°. The other one with the bigger orientation difference occurs at the points whose orientations are near to the angle of $-\pi/2$ or $+\pi/2$ when defining the orientation field in the range $[-\pi/2, +\pi/2]$, while 0 or $\pi$ when the orientation field is defined in the range $[0, \pi]$.

(3) The orientation variations of the points on the closed curve are gradual, that is, the absolute values of orientation differences between adjacent elements are less than $\pi/2$, except for the transition point with big orientation differences. Fig. 2 shows the variation of ridge orientations around singularity, in which the blue line represents the inside ridge and red ellipse labels the points where a sudden orientation change occurs.

To capture the sudden change of orientation, and extract singular points more reliably, we made some improvements on the Poincaré index method. Firstly, modulate the third part of (18) to $\delta(k)-\pi$, as

$$
A(k) = \begin{cases} 
\delta(k) & \text{if } |\delta(k)| < \frac{\pi}{2} \\
\pi+\delta(k) & \text{if } \delta(k) \leq -\frac{\pi}{2} \\
\delta(k)-\pi & \text{otherwise}
\end{cases}
$$

Moreover, the following constraints proposed are complementary to (21) when calculating the sum of orientation differences along the closed curve.

(1) Count the number of transition points from positive orientation to negative orientation and from negative orientation to positive orientation along the closed curve. If there is only one, continue to calculate the Poincaré index value. Otherwise, this point being handled will be treated as a normal point.

(2) Count the number of difference whose absolute value is greater than or equal to $\pi/2$ among $N_g$ differences on the closed curve. If the number is greater than one, then the point is treated as a normal point.

Formula (21) and the two additional constraints make Poincaré index more robust to noise and suitable for capturing the transition point around singularity in orientation fields with different resolution. Although it still does not reliably handle very poor quality fingerprints with cracks and scars, dry skin and so on, it is well enough as a step of our algorithm to guarantee the accuracy of detected singularities. Fig. 6 shows the distribution of singularities detected by improved Poincaré index, but before clustering. Red point represents delta point and the blue one represents core point. The detected delta and core points concentrate densely into red and blue areas, respectively.

Through the analysis for experimental result, it can be found that the distribution of singularities detected by this method has characteristics as below:

(1) The number of singularities which can be detected in singular region is relatively fixed, which will not increase as the number of calculus curve increases, as shown in Fig. 6. And singularities centralize in the inner part of singular region. For those poor quality fingerprint images, as long as the singular region is not polluted heavily by noise, the number of singularities detected is fixed as well.

(2) The collections of genuine core and delta points are relatively independent and have no intersection, while in the noisy area, both false cores and spurious deltas are detected and they overlap seriously.

![Fig. 6. Distribution of singular points extracted based on the improved Poincaré index, where red point represents delta point and the blue point represent core point. (a) and (b) are the results of high quality fingerprint image; (c) and (d) are the results of poor quality fingerprint image; (a) and (c) are the results detected in pixel orientation field; (b) and (d) are the results detected in block orientation field; (e) (f) (g) and (h) show the magnified views of labeled areas of (a), (b), (c) and (d), respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)](image-url)
The properties above, which can be viewed as the intrinsic features of fingerprints when using improved Poincaré index, will be exploited to further remove some spurious singularities in the next step of our algorithm.

4. Framework for multi-resolution orientation fields to detect singular points

In order that a singular points detection algorithm gracefully handles noise in a poor quality fingerprint as well as achieves an accurate localization of singularities, the detection should not only consider local features but also use global information [27]. And it is also important to reduce the dependence on single orientation field, prone to be affected by noise. To satisfy these conflicting requirements of an accurate and reliable localization, we proposed a framework of multi-resolution orientation fields to detect singular points, based on wavelet fundamental functions and Sampling theorem. Many other methods based on orientation field maybe benefit from this multi-level orientation fields framework.

4.1. Proposed multiresolution model to detect singularities

Multiresolution analysis [28], also called multi-scale analysis, is concerned with the representation and analysis of signals (images) at more than one resolution. In other words, multi-resolution analysis analyze the signals (images) at different frequencies with different resolutions, good space resolution and poor frequency resolution at high frequencies, while good frequency resolution and poor space resolution at low frequencies. The appeal of such an approach is obvious: features that might go undetected at one resolution may be easy to spot at another. This is the simple idea of multiresolution, and was realized by wavelet basis functions [29] as shown below:

$$\psi_{a,t}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t - \tau}{a} \right), \quad a > 0, \quad \tau \in \mathbb{R}$$ (22)

where $a$ is called scale factor, $\tau$ is called shift factor. Varying the values of $a, \tau$ is equivalent to scaling and shifting the basis function, then a band of wavelet basis functions is obtained. Simultaneously, a two dimensional scale-shift plane will form by varying the values of $a, \tau$, and it is, in some sense, the two dimensional space-frequency-domain plane. So wavelet transform can analyze signals (images) at space and frequency domains simultaneously by convolving with signals (images) using these basis functions. Furthermore, in a way, reciprocal of scale factor $a$ corresponds to frequency notion. Seen from (22), when $a$ is large, the width of space window is large thus space resolution is low, the width of frequency window is small thus frequency resolution is high, wavelet transform can extract low frequency information from images; on the contrary, when $a$ is small, the width of space window is small thus space resolution is high, the width of frequency window is large thus frequency is low, wavelet transform can extract high frequency information. A series of value $a$ can represent different frequencies, and $\tau$ is used to establish the region to analyze.

Fingerprint is a kind of texture image which has multi-resolution characteristics as mentioned in Section 2. The curvature will become lower and lower as the direction ranges from the singularity region to the outside. The singularity region corresponds to the region with highest frequency, the further the position is away from the singularity, the lower the frequency will be. Different frequency information is predominantly reflected in the orientation fields of fingerprints. Orientation fields with relatively high resolution obtained by using small block exhibit a sudden orientation change of the innermost ridges the most obviously, while orientation fields with less high resolution reflect sudden orientation change of relatively outside ridges the most conspicuously. Fingerprint singular points detection based on the Poincaré index just utilize the information of sudden orientation change in the neighborhood of singularities to localize singularity and establish its type. As analyzed above, for a fingerprint, the positions of sudden orientation changes in orientation fields of different resolutions are different, thus, we can get different positional singularities using this approach, and the correlation among them lend itself to accurate and reliable detection of singularities.

In this paper, we further develop the study in the literature [30] and propose a model to extract multi-resolution information from fingerprints based on multi-resolution analysis by means of giving a concrete implementation of wavelet basis function in singular point detection. In the 1D wavelet basis function $\psi_{a,t}(t) = (1/\sqrt{a})\psi((t-\tau)/a), a > 0, \tau \in \mathbb{R}$, $a$ is a scale factor, in some sense, $1/a$ implies the meaning of frequency, $\tau$ is a shift factor which is used to determine the position of image analysis. In the presented model, $a_i$ is used to represent the $ith$ level window, $W_i$ and $\tau_i$ are defined as the block size and shift interval of window $a_i$, respectively. In this paper, we adopt three level window scheme and assign the block size of the first level window, $W_1$, the value of 8, shift interval $\tau_1 = 2$. Thus, space resolution of this window is high, and can be used to extract or retain high frequency

![Fig. 7. Schematic diagram of block positions under different block sizes, different shift amounts.](image-url)
information in fingerprints. Note that size of the first level window depends on the various contexts in which this algorithm is applied. When \( a_2 = 2 \), since the wanted frequency is half of the frequency extracted by the first level window, based on the Sampling theorem, the block size of window \( W_2 \) is set to 16, and the sampling frequency could be reduced by half, so the shift interval is set to \( \tau_2 = 4 \). The binary discrete scheme for \( W_1 \) is usually adopted to discretize continuous wavelet functions. When \( a_3 = 3 \), the extracted frequency is one third of that of the first window, let block size of window \( W_3 \) be equal to 24, in the same way, \( \tau_3 \) is set to 6, namely, \( W_i = (a_i/a_1) \times W_1 (i = 2, 3) \) according binary discrete scheme, \( \tau_i = (a_i/a_1) \times \tau_1 \) according to the Sampling theorem. It is worth mentioning that in the case of two dimensions (2D), the window shifts along the horizontal and vertical directions, respectively.

For the window \( W_1 \) of level \( a_1 \), corresponding orientation field can reflect the sudden orientation changes of the innermost ridges in singular region. As shown in Fig. 7, the innermost ridges of singular points region have the highest curvature, and the corresponding area, in which abrupt changes of direction occur, is very small, therefore, block size of small window \( W_1 \) is suitable for this small area, while abrupt direction changes in this area will be smoothed when using big block. For ridges which are just next to the innermost ridges, since the region where abrupt direction changes occur is becoming large, block size of big window is more suitable for this region, while orientation fields obtained by small window will miss the information of orientation abrupt change in this area, can only reflect the gradual change of orientations of this area. In short, specific block size is suitable for extracting singularities with specific curvature. The position of block also influence extraction of the information of abrupt orientation change, further influence the extraction of singularities. The positions of singular points extracted from orientation fields obtained with different block shifts but the same block size are different. They concentrate into a small area in the singular points region, as shown in Fig. 8. There exists an optimal block position for each block size, as shown in Fig. 4, explained in Section 2. In this paper, we attempt to approach the optimal block position through several time translations along \( x \) and \( y \) directions, the interval of which is assigned based on Sampling theorem. The block shift can adequately mine the potential information of orientation fields under different block size for a fingerprint, thus can reduce missing detection for singularities.

Different block sizes have the capability to detect singularities of different resolutions. Singular point detected on level \( a_1 \), whose position is the most accurate among the three levels, is used to establish the final localization of corresponding singular point, and hence the precision of the detected singular point is 8 pixels, depending on the block size of window \( W_1 \). In addition, to achieve reliable detection, for core points, we use the linear relation among cores extracted from different resolution orientation fields to distinguish genuine cores, as shown in Fig. 9; for delta points, since delta point is a confluent point of three different flowlike directions, deltas detected by different block sizes do not satisfy the linear relation, but satisfy overlapped relation, to some extent, and this information is useful for reliable delta detection. Besides, for the purpose of further improving the noise immunity as well as reducing undetected ratio of delta points, we incorporate ridge coherence information into deltas detection. As according to the property that delta is a confluent point, using ridge coherence information is likely to be the most valid measure for delta point detection. The coherence information tendency of ridges has advantages to reliably detecting deltas. Coherence information of ridges is computed by using Gaussian–Hermite Moment, proposed by Shen [31,32], is a smoothed orthogonal moment suitable for extracting characters from poor quality images.

Coherence image of a fingerprint is computed using the method of Wang and Dai [33]. Fig. 10 shows the coherence image of a fingerprint (hereinafter referred to as c image), in which white color represent that the value of coherence is 1 and black is 0. Based on the analysis for the coherence image, a summary is presented below.

1. **Normal area**: The ridges are parallel, and its coherence is better, thus \( c \) value is bigger. As is shown in Fig. 10(d).
2. **Single singularity area**: Singular area has the highest ridge curvature, and the worst ridge coherence. What is more, with the distance becoming farther away from the singularity, the ridge curvature becomes lower and lower, and ridge coherence becomes better and better. Therefore, the \( c \) value in ideal singular area is a local minimum, the three dimensional shape assumes an inverted cone, and its vertex is just the position of singularity, as illustrated in Fig. 10(c).
(3) **Noisy area**: As for the area with big noises (i.e., area with most pseudo-points not eliminated in the multi-resolution orientation fields), no matter the window size is large or small, the value of $c$ over the whole area is smaller. As for the area with small noises, when the window size for computing $c$ becomes larger, the impact resulted from the noises will be weakened, thus both cases cannot form local minimum, as illustrated in Fig. 10(g).

(4) **Special case**: As for the special case that two singularities are close to each other, such as the tented arch and whorl, $c$ values do not always have gradually incremental tendency along the direction from one singularity to another (as shown in Fig. 10(e)). Therefore, this area cannot form prominently local minimum value (as illustrated in Fig. 10(f)). The detection of local minimum value either results in the miss of true singular points, or leads to the false detection. In some low quality fingerprints, due to the impact of noises, there are same problems around the single singularity.

To sum up, through detecting the tendency of $c$ values of candidate singularities’ neighborhood, we can determine whether it is true. Especially for delta point, since it is a confluent point, the case that two deltas are very close to each other scarcely occurs. In addition, the detection for $c$ should have directional feature, i.e., true singularity might not be the rigid local minimum because of noises. In our method, if there is a candidate delta that the $c$ value in the most directions of it has gradually incremental tendency, then it will be regarded as a true singularity.

Specifically, for each candidate delta, calculate value $c$ of its circular neighborhood whose radius is $6\tau$ ($\tau$ is the average ridge distance). Then segment the $c$ image into 32 sectors and a circle using the template showed in Fig. 11 and calculate their average $c$ values. Finally, compare the average $c$ values of three sectors on each direction among 16 directions. If $c$ increases from inside sector to outside sector, then label the direction as valid one. For a certain candidate delta, if the number of valid direction is more than 10, it is labeled as true singularity. Otherwise, label it as false delta. If sectors on some directions of a candidate singularity are out of fingerprint boundary, take the remaining directions into consideration, and calculate valid directions on these directions.

**Fig. 9.** Linear relationship of the cores detected from orientation fields with different resolutions. Blue, red, green square denote core points extracted from orientation fields obtained by using small, medium, large block sizes, respectively. And this relationship can be used to achieve reliable detection of cores. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Fig. 10.** Coherence image of a fingerprint. (a) Original image, (b) coherence image, (c) single singularity region, (d) normal area, (e) tendency of the $c$ value of the pixels on the red line, (f) the region of double singularities, (g) noisy region.
Add that in our experiment we use the window with size of \((4\tau + 1) \times (4\tau + 1)\) to calculate \(c\) of every pixel.

4.2. Algorithm description

Our proposed algorithm mainly has the following steps, as shown in Fig. 12.

1. Segment original fingerprint image.
2. Detect singular points under different block size and block shift amount.
3. Establish singularities accurately and reliably based on relative position relation of singularities extracted from different resolution orientation fields and ridge coherence information.

After the segmentation operation, valid regions are obtained on which we compute orientation fields with different block sizes and block shift amounts to detect singularities.

1. Compute the block positions. Block size is classified as \(m\) levels, let \(a_i\) denote the \(i\)th level, block sizes of various level windows are represented as \(W_1 \times W_1, W_2 \times W_2, \ldots, W_m \times W_m\) (In our experiment, we set \(W_1 = 8, W_i = W_1 \times i, i = 1, \ldots, m\)), respectively. The interval and the number of block shift under block size \(W_i\) are \(\tau_i\) and \(N_i (i = 1, \ldots, m)\), where \(\tau_i = i \times \tau_1\) (In our experiment, \(\tau_1 = 2\), \(N_i = W_i / \tau_i - 1\). For each level of block size \(W_i \times W_i\), let block position shift along the positive direction of \(x\) axis and negative direction of \(y\) axis for \(N_i\) times and \(\tau_i\) pixels for each time, respectively. Thus, for a certain block size \(W_i \times W_i\), that will generate \((N_i + 1) / \tau_i\) block positions, amounting to \(m \times (N_i + 1) \times (N_i + 1)\) block positions for \(m\) level block sizes.
2. Estimate orientation fields. For each block of fingerprint mentioned above, we estimate its orientation using a method proposed in [24].
(a) Compute the gradients of the x direction and y direction at each pixel belonging to a certain block.

(b) Estimate the local orientation of each block using gradients of all pixels in the block.

In order to ameliorate the accuracy of singularity localization and reduce missing singularities ratio, we do not smooth the orientation fields. (3) Detect singular points under different block sizes. Now assume that Singular\(_{i,j}\) is the singularity set extracted from orientation fields with different block sizes and block shift amounts, where \(i\) represents the level of block size \((i = 0, \ldots ,m-1)\), \(j\) denotes different shift numbers \((j = 0, \ldots ,N_i + 1)\). Singularity detection in a certain block size can be performed as follows.

(a) Compute the Poincaré index value of each block \((i,j)\) of orientation field and determine the interesting point of a block as core or delta according to its Poincaré index value. The size of mask used to calculate Poincaré index value is \(3 \times 3\) (see Fig. 13). The interesting point is marked as central point of a block.

(b) For each shift and each block size, the singularities determined by Poincaré index compose singularity set Singular\(_{i,j}\).

(c) Merge the singularities detected from orientation fields obtained by many shifts but the same block size, and then cluster singularities of each block size according to their types, respectively, then remove the clusterings which contain more singularities of the other type than its own type in the clustering region. Among the remaining clusterings, we compute the centroids of clusterings where the number of singularities is larger than threshold, \(\text{(threshold), represents threshold of the }i\text{th block size, so accurate and relatively reliable candidate singularity set \{Singular\(_{i,j}\}\} i\text{ is the level of block size, }j\text{ denotes the }j\text{th detected singularity, }i = 0, \ldots ,m-1, j = 0, \ldots ,\text{j}}\) is obtained by the steps described above.

After getting the singularity set Singular\(_{i,j}\) of different block size \((m=3)\), we distinguish between the genuine singularities and the spurious singular points according to the correlation of positions of singularities extracted from three level orientation fields and treat the corresponding singularity detected by the smallest block size as the final true singularity. The steps and principle for singularity combination of three level block sizes are as follows.

(1) Detect cores

(a) For each core extracted based on block size \(W_m \times W_m\), check whether there exist cores of block size \(W_{m-1} \times W_{m-1}\) that overlap with this core. The overlapped conditions are as follows:

\[
\text{Singular}_{m,x}^i - \text{Singular}_{m-1,x}^i \leq \Delta x
\]

\[
\text{Singular}_{m,y}^j - \text{Singular}_{m-1,y}^j \leq \Delta y
\]

where \(\text{Singular}_{m,x}^i\) and \(\text{Singular}_{m,y}^j\) specify the x coordinate and y coordinate of the ith core point of block size \(W_m \times W_m\), respectively, \(\Delta x\) and \(\Delta y\) represent the threshold along the x direction and y direction, respectively. The following concepts have the similar meanings.

(b) If there exists cores of block size \(W_{m-1} \times W_{m-1}\) satisfying the above overlapped conditions, we will take each \(\text{Singular}_{m-1,y}^j\) as a new base point to determine the core \(\text{Singular}_{m,2,y}^i\) of the smallest block size \(W_{m-2} \times W_{m-2}\) satisfying the above overlapped conditions. Therefore, for one core of block size \(W_m \times W_m\), there may be many corresponding cores of block sizes \(W_{m-1} \times W_{m-1}\) and \(W_{m-2} \times W_{m-2}\) satisfying overlapping conditions. Among the overlapping cores of block size \(W_{m-2} \times W_{m-2}\), we establish a core whose coordinate can nearly construct a straight line with the coordinate of base core \(\text{Singular}_{m,2,y}^i\) of block size \(W_{m-1} \times W_{m-1}\) as well as \(\text{Singular}_{m,2,w}^i\). This core point will be treated as a true singularity and set the highest confidence value. Assume \(d\) denote the distance between \(\text{Singular}_{m,2,y}^i\) and a straight line determined by \(\text{Singular}_{m,2,w}^i\) and \(\text{Singular}_{m,2,w}^j\). We can get the following equations:

\[
d = \sqrt{(A \times \text{Singular}_{m-1,y}^i + B \times \text{Singular}_{m-1,y}^i + C)^2 + A^2 + B^2}
\]

\[
A = \text{Singular}_{m-2,y}^i - \text{Singular}_{m,y}^i
\]

\[
B = \text{Singular}_{m,y}^i - \text{Singular}_{m-2,y}^i
\]

\[
C = -\text{Singular}_{m,y}^i \times A - \text{Singular}_{m,y}^i \times B
\]

If \(d\) is less than \(\text{DisThreshold}\), the core \(\text{Singular}_{m-2,w}^i\) is viewed as a true core, where \(\text{DisThreshold}\) is set based on empirical data.

(c) If there are corresponding points which satisfy the overlapping conditions in three level block sizes, but do not satisfy approximate straight line relation, then we will consider the core of block size \(W_{m-1} \times W_{m-1}\) as a base point, which is used to find a new base point which meets the condition of maximum overlapping area in block size \(W_{m-1} \times W_{m-1}\), the true core refers to the point in block size \(W_{m-2} \times W_{m-2}\) which has the maximum overlapping area with the new base point, this step will be ignored if two cores of the highest confidence have already been extracted.

(d) If there are no points to meet above conditions, we divert to deal with another core of block size \(W_m \times W_m\).

(3) Detect deltas.

(a) Take each delta in block size \(W_m \times W_m\) as a base point to check whether there exist deltas of block size \(W_{m-1} \times W_{m-1}\) that meet above overlapping condition. If there exist the corresponding points of block size \(W_{m-1} \times W_{m-1}\) which meet the demand, we will find the point which has the maximum overlapping area and take it as a new base point, which is used to find the point with the maximum overlapping area in block size \(W_{m-2} \times W_{m-2}\), this point is regarded as a true delta and is set the highest confidence value.

(b) For each delta of the level \(W_1\), we distinguish the genuine singularities according to ridge coherence change extracted by Gaussian–Hermite Moment. This step will be skipped if two deltas of the highest confidence have already been detected.

| i+1, j-1 | i+1, j | i+1, j+1 |
| i, j-1 | i, j | i, j+1 |
| i-1, j-1 | i-1, j | i-1, j+1 |

Fig. 13. The Poincaré index mask.
5. Experimental results

To test the performance of our algorithm, we carried out experiments on NIST Special Database 4 (NIST-4) [34] and public fingerprint databases FVC02 DB1 and DB2 [35]. The NIST-4 contains 2000 pairs of 8-bit gray scale fingerprint images with the size of $512 \times 512$. Many fingerprint images in NIST-4 suffer from creases, scars and smudges. Both DB1 and DB2 contain 800 fingerprints, namely, 100 fingers and eight prints for each finger.

The singular points of the fingerprints in NIST-4 are manually labeled beforehand to obtain ground truth. For a ground truth singular point $(x_0, y_0)$, if a detected singular point $(x, y)$ satisfies $|x - x_0| < W_1$ and $|y - y_0| < W_1$ (in our experiment, $W_1$ is set to 8), it is said to be truly detected and, otherwise, it is called a miss. The detection rate is defined as the ratio of truly detected singular points versus all ground truth singular points. The miss rate refers to the ratio of the number of missed singular points to the number of all ground truth singular points. The false alarm rate is defined as the number of falsely detected singular points versus all ground truth singular points. If all singular points of a fingerprint are truly detected and there are no spurious singular points, the fingerprint is regarded as being truly detected.

We perform singular points detection according to the algorithm described in Section 4.2. It should be mentioned that there exists a special type of fingerprint, the core and delta points of which are fairly close to each other, and only in high resolution orientation field these kinds of singularities are visible, as shown in Fig. 14. Hence, it is well known that detecting this type of fingerprint is much difficult. In our method, because the framework comprises three level orientation fields, some detected points with more clustering elements in high level orientation field are added to candidate singular set, and then the ridge coherence information method presented in Section 4.1 is taken to confirm whether these candidate singularities are true or false. This complementary step further utilizes the rich information provided by our framework and exhibits our advantage on dealing with this type of fingerprints, comparing with some methods depending on single resolution orientation field.

To give a comparative study, we first compare the proposed algorithm with Poincaré index-based as well as multi-resolution-based method [21]. The comparison results are listed in Tables 1, 2. Fig. 15 presents the detection results on some low quality fingerprints. From these results, it can be seen that the singular points were more robustly detected when using the proposed algorithm. We also have compared our method with two recent techniques reported to achieve a better performance than others, that is, Zhou’s [14] and Chikkerur’s [12] methods. These comparison results are listed in Table 3. The results of comparison illustrate that our method has more equivalent performance to Zhou’s method. This can be explained that our method not only combines global information and local information but also mines some new effective information to detect singularities. Because of the use of linear relation of cores detected in different resolution fields and ridges coherence change of singularities (cores + deltas), our algorithm is more robust to noises. Determining the final positions of singularities in the highest resolution orientation field makes our algorithm reasonably accurate. At last, it is worth noting that the proposed method have identical computational complexity with some conventional methods $O((M \times N) \times W \times L)$, where $M \times N$ denotes the size of image, $W$ refers to the window size when computing orientation fields (three scales 8, 16, 32), $L$ means that three different resolution orientation fields are utilized.

6. Conclusion

This paper suggests a model to detect singular points, which incorporates the Poincaré index feature, multi-resolution characteristic and ridges coherence information into singularities detection. The traditional Poincaré index is ameliorated to make it more robust and suitable for multi-resolution orientation fields. Relationship of singularities detected in different resolution orientation fields and ridges coherence information of candidate singularities are used to properly erase noise-introduced singular points. Singularities positions are determined in high resolution orientation field. This makes our method more robust and precise than other methods only using local information. Besides, another contribution of this study lies in that the presented multi-resolution orientation field model, as a

Table 1
<table>
<thead>
<tr>
<th>Items</th>
<th>Proposed</th>
<th>Zhou’s</th>
<th>Tico’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singular points (cores + deltas)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detection rate</td>
<td>96.14</td>
<td>96.10</td>
<td>76.53</td>
</tr>
<tr>
<td>False alarm rate</td>
<td>4.39</td>
<td>4.30</td>
<td>30.67</td>
</tr>
<tr>
<td>Cores</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detection rate</td>
<td>96.85</td>
<td>95.78</td>
<td>90.27</td>
</tr>
<tr>
<td>False alarm rate</td>
<td>3.14</td>
<td>2.27</td>
<td>10.78</td>
</tr>
<tr>
<td>Deltas</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detection rate</td>
<td>94.01</td>
<td>96.68</td>
<td>55.49</td>
</tr>
<tr>
<td>False alarm rate</td>
<td>7.63</td>
<td>9.97</td>
<td>80.20</td>
</tr>
<tr>
<td>Fingerprints</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correct rate</td>
<td>89.13</td>
<td>88.88</td>
<td>58.50</td>
</tr>
</tbody>
</table>

Table 2
<table>
<thead>
<tr>
<th>Items</th>
<th>Proposed</th>
<th>Zhou’s</th>
<th>Tico’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singular points (cores + deltas)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detection rate</td>
<td>94.57</td>
<td>94.51</td>
<td>50.81</td>
</tr>
<tr>
<td>False alarm rate</td>
<td>9.67</td>
<td>9.60</td>
<td>96.83</td>
</tr>
<tr>
<td>Cores</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detection rate</td>
<td>96.23</td>
<td>95.95</td>
<td>65.38</td>
</tr>
<tr>
<td>False alarm rate</td>
<td>8.60</td>
<td>8.45</td>
<td>52.94</td>
</tr>
<tr>
<td>Deltas</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detection rate</td>
<td>90.51</td>
<td>90.88</td>
<td>34.75</td>
</tr>
<tr>
<td>False alarm rate</td>
<td>12.31</td>
<td>12.54</td>
<td>187.80</td>
</tr>
<tr>
<td>Fingerprints</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correct rate</td>
<td>81.50</td>
<td>81.25</td>
<td>32.32</td>
</tr>
</tbody>
</table>

![Fig. 14. Detection results of our proposed method for singularities which are fairly close.](image)
framework, may be directly exploited in other techniques relying on orientation field.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under No. 61070097, by the Research Fund for the Doctoral Program of Higher Education under Grant No. 2010013110021 and by the Natural Science Foundation of Shandong Province under Grant No. Z2008G05.

References


Dawei Weng was born in 1984, received B.S. degree at Shandong University of Finance in 2006 and received M.S. degree at Shandong University in 2009. He is currently working toward the Ph.D. degree at School of Computer Science and Engineering, Beihang University. His research interests are fingerprint recognition, image processing and machine learning.

Yilong Yin at present is the Director of MLA Group and Professor of Shandong University. He received his Ph.D. degree at Jilin University, China, in 2000. From 2000 to 2002, he worked as a Post Doctor in the Department of Electronic Science and Engineering, Nanjing University, China. His research interests include machine learning, data mining and biometrics.

Dong Yang was born in December 1985, he received his B.S. degree at Shandong University in 2007. Now he is a master student in School of Computer Science and Technology, Shandong University. His research interests are pattern recognition and image processing.